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Fig. 1.

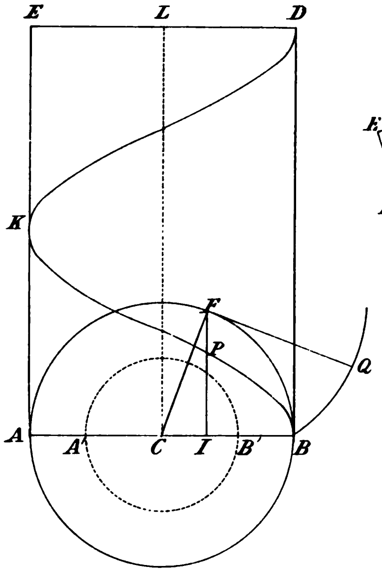


Fig. 3.

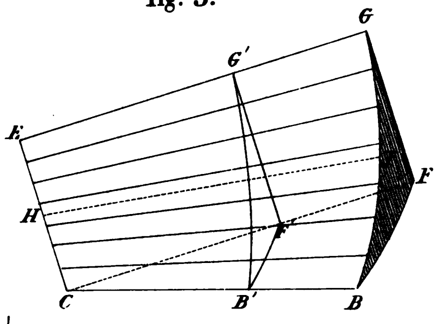


Fig. 4.

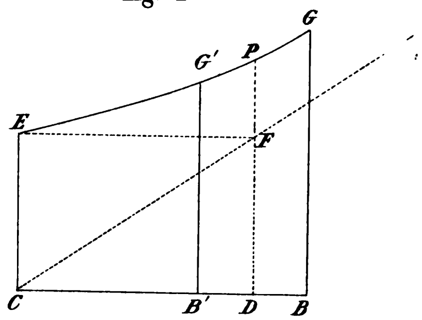
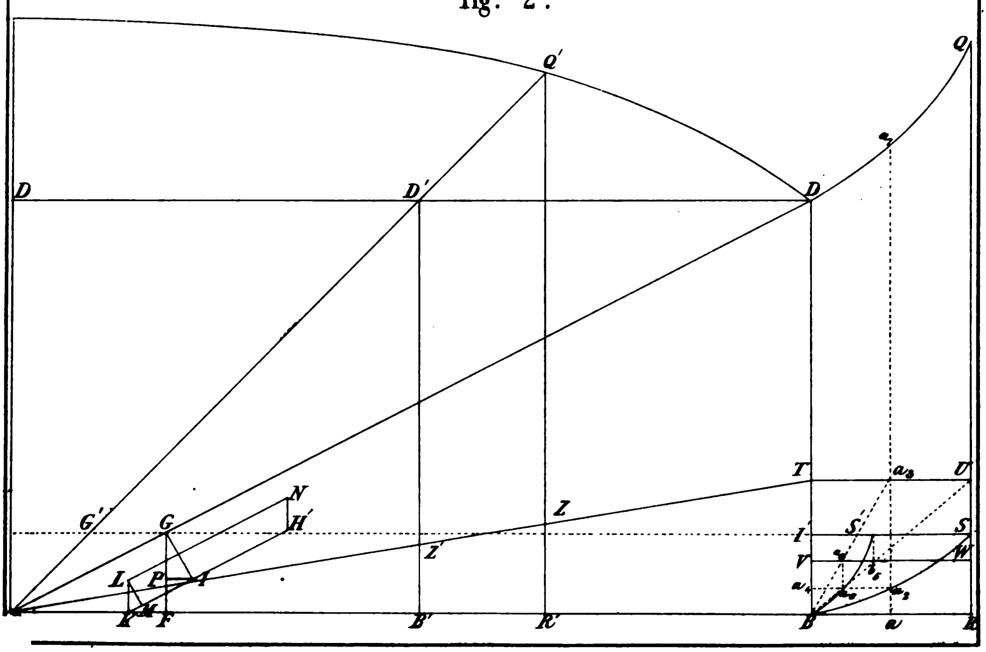


Fig. 2.



THE

SCREW PROPELLER;

AN INVESTIGATION OF ITS

GEOMETRICAL AND PHYSICAL PROPERTIES,

AND ITS APPLICATION TO

THE PROPULSION OF VESSELS.

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LONDON:
PUBLISHED BY WHITTAKER AND CO.,
AVE-MARIA-LANE.

1851.

LONDON:
PRINTED BY S. MILLS, CRANE-COURT, FLEET-STREET.

TO

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MY DEAR SIR:

I beg to inscribe the following pages to you, in grateful remembrance of the kind encouragement you have uniformly given me in scientific pursuits. That you may long enjoy the reputation which your important researches have justly obtained ,is the sincere wish of

Your ever faithful friend,

And humble servant,

ROBERT RAWSON.

PORTSMOUTH DOCK-YARD,
November, 1850.

ERRATA.

Page	4, line 14 from the top,	for	x, y, x, read x, y, z.
,,	15, line 38	,,	$p - \frac{2\pi v}{u}$ read $p - \frac{2\pi v}{w}$
,,	15, line 39	,,	u, read w.
,,	15, line 41	,,	*vessel*, read *screw*.
,,	15. line 41	,,	*screw*, read *vessel*.
,,	20, line 26	,,	H, read H'.
,,	30, line 6	,,	s, read S.
,,	36, line 29	,,	Υ'_{ϖ}, read Υ_{ϖ}.
,,	36, line 30	,,	$\Upsilon_{\dot{\varpi}}$ and Υ'_{ϖ}, read R_{ϖ} and R'_{ϖ}.
,,	36, line 33	,,	ditto ditto.
,,	56, line 15	,,	B', read B.
,,	56, line 16	,,	ditto.
,,	61, line 13	,,	Z, read z.
,,	61, line 19	,,	P, read p.
,,	61, line 28	,,	*problem*, read *problem* (10).
,,	65, line 11	,,	$\frac{d}{dx}$ read $\frac{d}{dx'}$

PREFACE.

THE increasing national importance of the subject discussed in the following pages, the limited information to be obtained from English publications on the purely scientific part, and the more limited mathematical investigations which have been made in the theory, must apologize for my presuming to lay the following pages before the public.

I cannot flatter myself that the following exposition of the subject of Screw Propulsion is complete, or that the arrangement is the best that might have been adopted; being the results of my consideration of the subject, in the order in which they presented themselves to my own mind, rather than according to a preconceived arrangement; but, although the investigations are not so complete as I could have desired to make them, still I trust that the Mathematician, Naval Architect, and Engineer will find in them much that is new, and at the same time capable of standing the test of accurate mathematical examination.

The following are some of the more important conclusions arrived at:

The discovery of the *surface of vanishing pressure*, detailed in the second chapter, is important, as it affords an explanation of some curious phenomena which have been observed in the practice of Screw Propulsion.

The investigation of the moment of inertia of the screw blade, as given in problem (5), chapter ii., and its application to the determination of the accelerating forces acting on the blade of the Screw Propeller, are entirely new, as well as the results obtained, in the notes at the end of the second chapter, on a subject highly interesting both to the practical Naval Architect and the theoretical inquirer. The formulæ for the area, pitch, radius of gyration, and moment of inertia of the screw propeller blade, and the table, computed at considerable labour, for their application in practice, will, it is hoped, be found useful to practical men.

Among the results which have been brought out in the following investigation of the properties and action of the screw propeller, the singular and important relation which is expressed by equation (7), page 33, will be found very useful in determining the *quality* of the screw, and of the engine which is used to turn it; as it is not always the case that the largest and most powerful engine is the best adapted to the Screw Propeller.

The engineer will readily see the importance of the fact, that a moment of force is absolutely necessary to produce a motion of rotation, and that this moment of force consists of two independent elements, the force, and the distance from the point of its application to the centre of rotation; therefore, the moment of an engine should be regarded, in the estimating of its quality for this purpose, and not the absolute force as measured by

the indicator, or the pressure on the piston. An engine which may have a large force on the piston, and a small moment, is not advantageously adapted to give rotatory motion to the Screw Propeller.

The peculiar form of the Screw Propeller blade gives rise to the resolution of the forces acting upon it into two directions; one the direction of the vessel's motion, and the other to turn the screw round its axis. These two resolutions give two equations, each of which contains the same indeterminate integral, which depends on the (at present) unknown law connecting the accelerating resistance of the water with the normal velocity of the screw blade; by means of these two equations the unknown integral can be eliminated, and the result is an important relation between the pressure of the water on the blade of the screw in the direction of its axis, and the moment of force which is exerted by the engine when the screw and the vessel have obtained a uniform motion.

Equation (5), page 37, is worthy of attention, as expressing a formula which enables the Naval Architect to measure with certainty the *quality* of any vessel, and thereby to free himself from the many perplexities which arise from the unequal skill of commanders in their management at sea.

The subject of inquiry in the third chapter is also new, and, I trust, is rendered as simple as the nature and difficulty of the inquiry will permit. It is scarcely necessary to state how much I am indebted to the important labours of Professor Hodgkinson on the subject of this chapter. The fourth chapter contains an investigation of the different forms which have been proposed for Screw Propellers, and of the forces which act upon any form of blade, and also an inquiry into the peculiar surfaces which possess the important property of a *surface of vanishing pressure*.

The use of the *Pitch-compass*, which was invented with a view to facilitate the admeasurement of the pitch of the screw, is described in the note appended to the fourth chapter. The instrument was originated to obviate the difficulty experienced in practice, in ascertaining the pitch of the screw by the methods hitherto in use; for which purpose it is now adopted in Her Majesty's Dock-yard at this place. This is followed by an examination of the cause which operates to produce oscillations at the stern of a vessel propelled by the screw; wherein it is proved that Smith's screw blade is the only surface which will not produce such oscillations.

With this brief summary of the principal contents of these pages, I leave them to the candid consideration of the reader; and if I shall have succeeded in removing any of the difficulties which invest the examination of this increasingly interesting and important subject, I shall be amply repaid for the labour of the inquiry.

ROBERT RAWSON.

Portsmouth Dock-yard, November, 1850.

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THE SCREW PROPELLER.

CHAPTER I.

DESCRIPTION OF THE HELIX, OR THE SPIRAL OF ARCHIMEDES.

$A\ B\ E\ D$ (fig. 1) is a section of a cylinder, perpendicular to the diameter $A\ B$; $A\ F\ B$ is a circular section at the end of the cylinder: if a chord-line be fastened at B, and wrapped once round the cylinder and fastened at D, the chord-line will trace upon the smooth cylinder the spiral of Archimedes or the common helix.

Draw $B\ B$, in fig. 2, equal to the circumference of the circle $A\ F\ B$, fig. 1; on this line, construct a rectangular parallelogram, $B\ D$, whose height is $B\ D$, fig. (1); join $B\ D$: then, it will be readily seen that the parallelogram $B\ D$ will be the development of the cylindrical surface, and the line $B\ D$ will be the development of the helix or spiral.

The curve $B\ K\ D$ in fig. (1) is the othographic projection of the spiral on the plane $A\,B\,D\,E$. This curve is constructed in the following manner: in the circumference $A\ B$ take any point F, and make $B\,F$ in fig. (2) equal to the arc $B\,F$ in fig. (1); draw $F\,G$ perpendicular to $B\,B$, and $F\,I$ fig. (1) perpendicular to $A\,B$; take $I\,P$ equal to $F\,G$; then P will be a point in the curve $B\ K\ D$;—continue this process, and any number of points may be obtained through which the curve $B\,K\,D$ may be drawn.

Definitions.—$C\,L$ perpendicular to $A\,B$ is called the axis of the cylinder.

The line $B\,D$, sector $F\,C\,B$ fig. (1), and $G\,F$, $\angle\ G\,B\,F$ fig. (2), are called the pitch, base, height, and angle respectively of the spiral or helix $B\,P$.

It will be convenient to denote these magnitudes by the following symbols, which will be invariably adopted:

r = radius of cylinder.
p = pitch.
β = angle of sector $F\,C\,B$ or base of the helix.
A = angle of helix.
h = height of spiral, or length of screw.

PROBLEM I.

To find the Equation to the Helix.

Let x, y, and z be the rectangular co-ordinates of any point P in the helix fig. (1). Origin being at C, the axes of x and z are CB and CL, to which the axis of y is perpendicular.

We have

$$y = r \sin. \beta \text{ and } x = r \cos. \beta$$

$$\text{But, } BF = r\,\theta, \text{ and } GF = z = BF.\tan. A \text{ fig. (2).}$$

$$\therefore z = r\,\beta.\tan. A. \text{ or } \beta = \frac{z}{r \tan. A}$$

$$\therefore y = r \sin. \left(\frac{z}{r \tan. A}\right)\ x = r \cos. \left(\frac{z}{r \tan. A}\right) \text{ and } x^2 + y^2 = r^2 \quad . \quad . \quad (1).$$

These equations are the projections of the spiral on the planes of $y\,z$, $x\,z$, and $x\,y$.

The projections of the helix on the planes $y\,z$, $x\,z$, $x\,y$ are the curve of sines, co-sines, circle, respectively.

Cor. (1).—The equations to a tangent to the helix at a point whose rectangular co-ordinates are x', y', and z' are

$$y - y' = \frac{d\,y'}{d\,z'}(z - z') \text{ and } x - x' = \frac{d\,x'}{d\,z'}(z - z')$$

By differentiating (1) we shall obtain the values of $\frac{d\,y'}{d\,z'}$ and $\frac{d\,x'}{d\,z'}$ as follows:

$$\frac{d\,y'}{d\,z'} = \cot. A.\cos. \left(\frac{z'}{r \tan. A}\right) \text{ and } \frac{d\,x'}{d\,z'} = -\cot. A \sin. \left(\frac{z'}{r \tan. A}\right)$$

$$\therefore y - y' = \frac{x'}{r \tan. A}(z - z') \text{ and } x - x' = -\frac{y'}{r \tan. A}(z - z')$$

$$\text{and } y - y' = -\frac{x'}{y'}(x - x') \quad . \quad . \quad . \quad (2)$$

which are the projections of the tangent on the planes of $y\,z$, $x\,z$, and $x\,y$, respectively.

The last of equations (2) shows that the projection of a tangent to the helix on the plane $x\,y$ is a tangent to the circle $A\,F\,B$ fig. (1); this is obvious from the geometrical properties of the cylinder.

Cor. (2).—Put X, Y for the rectangular co-ordinates of the point where the tangent to the helix intersects the plane of $x\,y$. Then from equations (2) we shall have the values of X and Y when z is put equal to nothing.

$$Y = y' - \frac{x'\,z'}{r \tan. A} = r \sin. \left(\frac{z'}{r \tan. A}\right) - \left(\frac{r\,z'}{r \tan. A}\right)\cos. \frac{z'}{r \tan. A}$$

$$X = x' + \frac{y'\,z'}{r \tan. A} = r \cos. \left(\frac{z'}{r \tan. A}\right) + \left(\frac{r\,z'}{r \tan. A}\right)\sin. \left(\frac{z'}{r \tan. A}\right) \quad . \quad . \quad (3).$$

We have, $y' Y + x' X = r^2$; from the 1st of equations (3).

And the second shows that the intersection of the tangent to the helix with the plane $x y$ is the involute of the circle $A F B$ fig. (1).

Hence, to find a point in the involute corresponding with F in the circle, draw the tangent $F Q$ and make $F Q$ equal to the arc $F B$, then Q is a point in the involute.

Cor. (3).—The pitch and length of the helix are readily obtained from fig. (2).

$$p = B B. \tan. A = 2 r \pi. \tan. A \qquad \ldots \ldots \qquad (4).$$

where $\pi = 3{\cdot}1416$, &c. &c.

Hence, the pitch is proportional to the tangent of the angle of screw.

$B D = \sqrt{4 r^2 \pi^2 + p^2}$ = length of helix to the height of the pitch.

$B G = \sqrt{r^2 \beta^2 + h^2}$ = length of helix to any height h.

Since $\beta = \frac{2 \pi h}{p}$ we have

$$L = B G = \frac{h}{p} \sqrt{4 \pi^2 r^2 + p^2} = h \operatorname{cosec.} A$$

Cor. (4).—To find the angle which the tangent to the helix at F makes with the plane of $x y$. Call this angle ϕ

$$\therefore \tan. \phi = \frac{z}{F Q} = \frac{r \beta. \tan. A}{r \beta} = \tan. A$$

$$\therefore \phi = A \text{ a constant magnitude.}$$

DESCRIPTION OF A CONOIDAL SURFACE.

Let $B C F$ fig. (3) be an horizontal plane to which $E C$ is perpendicular, and $B G$ be any curve line situated in space; if the straight line $E G$ move parallel to the plane $C B F$ from $C B$ to $E G$, so that one extremity of it is on $B G$ and the other on $C E$, the surface $C B G E$ is called a conoidal surface.*

By assuming various curved lines for $B G$ we may obtain a variety of conoidal surfaces.

If the line $B G$ be a circle perpendicular to the plane $B C E$, the surface $C B G E$ is Wallis's cono-cuneus.

If the line $B G$ be the helix, $C B$ the radius of the cylinder on which the helix is wrapped, the surface $C B G E$ is called the screw-blade which is extensively used in Her Majesty's navy for propelling vessels.

The screw-blade here described is known by the name of Smith's screw, who first applied it effectively to propel vessels. Gregory, in his elegant

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* The curve $B G$ is called the directrix, and the straight line $E G$ is called the generatrix.

work on Solid Geometry, calls this surface "skew-screw surface," (see page 168,) and observes that it is the "under surface of a spiral staircase, and is consequently one which is frequently presented to the eye, and is also easily constructed.

Professor Woodcroft's screw is distinguished from that of Smith, in not fixing the line $B\,G$ by any particular law, leaving the surface indeterminate, with a view to obtain the best possible surface for the screw-blade used to propel vessels. Hence, Woodcroft's screw is more general than Smith's, which is contained, as a particular case, in Woodcroft's, which appears not to have met with that consideration which an ingenious generalization amply deserves.

PROBLEM II.

To find the Equations to Smith's Screw-blade.

Let x, y, z be the rectangular co-ordinates of a point P in the surface $E\,C\,B\,G$ fig. (3), $B\,G$ is the helix described in Problem (1). $C\,B$, $C\,E$ are the axes of x, z, perpendicular to which is the axis of y, origin at C. If we call β' the angle of the plan corresponding to the height z, we shall have

$$\beta' = \frac{z}{r \tan. A}$$

$$\text{and } \frac{y}{x} = \tan. \beta'$$

$$\therefore y = x. \tan. \left(\frac{z}{r \tan. A}\right) \quad . \quad . \quad . \quad . \quad . \quad (1.)$$

which is the equation to the surface of Smith's screw-blade.

Cor. (1).—Draw $B'\,F'$ concentric with $B\,F$ and $F'\,G'$ parallel to $F\,G$; then the line $G'\,B'$ on the surface of the screw-blade will be an helix concentric with the helix $B\,G$, and having the same pitch.

$$\text{For, } x^2 + y^2 = C B'^2 = r'^2 \quad . \quad . \quad . \quad . \quad (2).$$

From (1) and (2) we shall obtain

$$y = r' \sin. \left(\frac{z}{r \tan. A}\right) \text{ and } x = r' \cos. \left(\frac{z}{r \tan. A}\right)$$

which are the equations to an helix concentric with $B\,G$, and if we put A' to represent the angle of the helix, and p' for the pitch, we shall have

$$r' \tan. A' = r \tan. A$$

$$\therefore 2 \pi r' \tan. A' = 2 \pi r \tan. A$$

$$\text{or } p' = p \quad \text{See cor. (3) to prob. (1).}$$

Cor. (2).—The length of the helix on the blade of the screw concentric with $B\,G$ is

$$y^2 = x^2 \beta^2 + h^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3).$$

where y denotes the length of $B'\,G'$ and x the length of $C\,B'$. The equa-

tion (3) suggests a mode of developing the blade of the screw in the following manner:

Make CB fig. (4) equal to the radius of the cylinder; draw CE, BG, perpendicular to CB, make CE equal to the height of the screw-blade, and BG equal to the length of the helix BG fig. (3).

Take a point B' in CB corresponding with B' in fig. (3), draw $B'G'$ parallel to BG, making $B'G'$ equal to the helix $B'G'$ in fig. (3), then G' will range in the curved line $EG'G$.

The area $EGBC$ fig. (4) is the area of the screw-blade $EGBC$ fig. (3); and the area $B'G'GB$ will be the area of the screw-blade included between the helices $B'G'$ and BG.

The area of this curve may be found by Simpson's method.

Cor. (3).—To construct the length of $B'G'$ in fig. (4).

In fig. (1) draw a circle concentric with BA corresponding with the point B' in fig. (4). Make BB' fig. (2) equal to the circumference of the circle $B'A'$ fig. (1); draw $B'D'$ perpendicular to BB', join BD', and draw HGI parallel to BB', cutting BD' in G'. Then BG' will be the length of the helix at the point B' in figs. (3) and (4): by continuing this process we may obtain as many points in the curve $EG'G$ as may be desirable to enable us to draw it with tolerable accuracy.

PROBLEM III.

To find the Nature and Area of the Curve Line $EG'G$ fig. (4).

If xy be the rectangular co-ordinates of any point G' origin at C,

We have by last Problem, equation (3).

$$y^2 = \beta^2 x^2 + h^2$$

$$\therefore \frac{x^2}{\left(\frac{h^2}{\beta^2}\right)} - \frac{y^2}{h^2} = -1 \quad . \quad . \quad . \quad . \quad (1).$$

If $x = o$, then $y = h$

If $y = o$, then $x = \frac{h}{\beta}\sqrt{-1}$

Hence the curve passes through E, but never meets the axis of x, in consequence of x being imaginary when $y = o$.

The curve is evidently an hyperbola whose vertex is E and centre C, the major and minor axes being

$$2\,CE = 2\,h \text{ and } 2\,CD = \frac{2\,h}{\beta}$$

Draw EF parallel to CD, and join CF; then CF will be an asymptote to the hyperbola.

To find the area we have from (1),

$$y = \sqrt{h^2 - \beta^2 x^2}$$

$$\therefore \int y\,dx = \int \sqrt{h^2\,\beta^2 + x^2}\,.\,dx + c$$

$$= h^2 \int \frac{dx}{\sqrt{h^2 + \beta^2 x^2}} + \beta^2 \int \frac{x^2\,dx}{\sqrt{h^2 + \beta^2 x^2}} + c$$

$$= \frac{h^2}{\beta} \cdot \log.\left(\beta x + \sqrt{h^2 + \beta^2 x^2}\right) + \frac{\beta^2 x}{2\beta^2}\sqrt{h^2 + \beta^2 x^2}$$

$$- \frac{h^2 \beta^2}{2\beta^3} \cdot \log.\left(\beta x + \sqrt{h^2 + \beta^2 x^2}\right) + c$$

$$= \frac{h^2}{2\beta}\log.\left(\beta x + \sqrt{h^2 + \beta^2 x^2}\right) + \frac{x\sqrt{h^2 + \beta^2 x^2}}{2} + c$$

This integral taken between the limits $x = r_1$ and $x = r_2$ will be as follows, if we put $A' =$ area between these limits :—

$$A' = \frac{h^2}{2\beta}\log.\left(\frac{r_1\beta + \sqrt{h^2 + r_1{}^2\beta^2}}{r_2\beta + \sqrt{h^2 + r_2{}^2\beta^2}}\right) + \frac{r_1\sqrt{h^2 + r_1{}^2\beta^2} - r'\sqrt{h^2 + r_2{}^2\beta^2}}{2} \quad . \quad . \quad (2).$$

Put $C B' = r_1$ and L_1 and L_2 for the length of the helix at the points r_1 and r_2,

$$\text{Then } A' = \frac{h^2}{2\beta}\log.\left(\frac{r_1\beta + L_1}{r_2\beta + L_2}\right) + \frac{r_1 L_1 - r_2 L_2}{2} \quad , \quad . \quad . \quad . \quad . \quad (3).$$

The hyperbolic logarithms are used in the above investigation ; if common logarithms be used in computation, the first term must be divided by the modulus of the common logarithms, viz., 434294484, or multiplied by 2·3025851.

The values of β, L_1, L_2, may be obtained from cor. (3), prob. (1).

PROBLEM IV.

To determine the Equation to a Tangent Plane at any point on the Surface of Smith's Screw.

Let x, y, z be the current co-ordinates of the tangent plane ; and $x'\,y'\,z'$ the co-ordinates of the point on the surface where the tangent plane is required.

The equation to the surface by (3) Prob. (2).

$$u = y' - x'\tan.\left(\frac{z'}{r\tan. A}\right) = 0 \quad . \quad . \quad . \quad (1).$$

$$\text{And } \frac{du}{dx'}(x - x') + \frac{du}{dy'}(y - y') + \frac{du}{dz'}(z - z') = 0 \quad . \quad . \quad . \quad (2).$$

is the equation to the tangent plane. (See "Gregory's Solid Geometry," page 168.)

By differentiating equation (1), we obtain—

$$\frac{du}{dx'} = -\tan.\left(\frac{z'}{r\tan.A}\right);\ \frac{du}{dy'} = 1, \text{ and } \frac{du}{dz'} = \frac{1}{r\tan.A}\cdot\frac{x'}{\text{Cos.}^2\left(\frac{z'}{r\tan.A}\right)}$$

$$\text{or } \frac{du}{dx'} = -\frac{y'}{x'} \text{ and } \frac{du}{dy'} = 1, \text{ and } \frac{du}{dz'} = \frac{y'^2 + x'^2}{x' r \tan.A}$$

Substitute these values in equation (2), and we shall have—

$$-\frac{y'}{x'}(x - x') + (y - y') + \frac{y'^2 + x'^2}{x' r\tan.A}(z - z') = 0$$

$$\text{or } -y'x + y'x' + x'y - y'x' + \frac{y'^2 + x'^2}{r\tan.A}(z - z') = 0$$

$$\therefore x'y - y'x + \frac{y'^2 + x'^2}{r\tan.A}(z - z') = 0 \quad . \quad . \quad . \quad (3).$$

which is the equation to the tangent plane.

PROBLEM V.

To find the Equation to the Normal at any point on the surface of Smith's Screw.

Let $x'\,y'\,z'$ be the point of the surface, and $x\,y\,z$ the current co-ordinates of the normal, the equations of which are

$$y - y' = \frac{\frac{du}{dy'}}{\frac{du}{dz'}}(z - z') \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1).$$

$$x - x' = \frac{\frac{du}{dx'}}{\frac{du}{dz'}}(z - z') \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

(See "Gregory's Solid Geometry," page 134.)

In these equations substitute the values of $\frac{du}{dy'}$ &c., &c., as given in last problem.

$$\therefore y - y' = \frac{x' r\tan.A}{x'^2 + y'^2}(z - z') \quad . \quad . \quad . \quad . \quad (3).$$

$$x - x' = -\frac{y' r\tan.A}{x'^2 + y'^2}(z - z') \quad . \quad . \quad . \quad (4).$$

These are the equations to the normal at a point whose co-ordinates $x'\,y'\,z'$ on the surface of the screw.

PROBLEM VI.

To find the Cosines of the Angles which the Normal makes with the Co-ordinates Axes, x, y, and z.

Let X, Y, Z be the angles which the normal makes with x, y, z respectively.

$$\therefore \cos. X = \frac{\frac{du}{dx'}}{\sqrt{\left(\frac{du}{dx'}\right)^2 + \left(\frac{du}{dy'}\right)^2 + \left(\frac{du}{dz'}\right)^2}}$$

$$\cos. Y = \frac{\frac{du}{dy'}}{\sqrt{\left(\frac{du}{dx'}\right)^2 + \left(\frac{du}{dy'}\right)^2 + \left(\frac{du}{dz'}\right)^2}}$$

$$\cos. Z = \frac{\frac{du}{dz'}}{\sqrt{\left(\frac{du}{dx'}\right)^2 + \left(\frac{du}{dy'}\right)^2 + \left(\frac{du}{dz'}\right)^2}}$$

See Gregory's Solid Geometry, page 135.

By substituting the values of $\frac{du}{dx'}$, &c., as given in Problem (4) in the above equations, we shall have

$$\left.\begin{aligned} \text{Cos. } X &= -\frac{y'}{(y'^2 + x'^2)\sqrt{\frac{1}{x'^2 + y'^2} + \frac{1}{r^2 \tan.^2 A}}} \\ \text{Cos. } Y &= \frac{x'}{(y'^2 + x'^2)\sqrt{\frac{1}{x'^2 + y'^2} + \frac{1}{r^2 \tan.^2 A}}} \\ \text{Cos. } Z &= \frac{1}{\sqrt{\frac{r^2 \tan.^2 A}{x'^2 + y'^2} + 1}} \end{aligned}\right\} \quad . \quad . \quad . \quad (1).$$

Hence the cosine of the angle which the normal makes with the axis of z is the same for every point in the helix concentric with the helix $B\,G$.

PROBLEM VII.

To find the Centre of Gravity of the Surface of Smith's Screw.

In figure (3) bisect $C\,E$ and $B\,G$ in the points H and I; then the centre of gravity of the surface $C\,E\,G\,B$ will be in the line $H\,I$. Since the surface is symmetrical about this line.

Let A' = surface of blade.

H = distance along the line $H\,I$ to the centre of gravity.

x = the distance along the line $H\,I$ to the variable helix $G'\,B'$.

$$\therefore A' H = \int L x \, dx \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1).$$

Where $L = B' G' = \sqrt{x^2 \beta^2 + h^2}$

$$\therefore L^2 = x^2 \beta^2 + h^2$$

$$\therefore x^2 = \frac{L^2 - h^2}{\beta^2}$$

$$\therefore x \, dx = \frac{L \, dL}{\beta^2}$$

$$\therefore A' H = \int \frac{L^2 dL}{\beta^2} = \frac{L^3}{3 \beta^2}$$

If we take the area between the limits $L = B\,G = L_1$ and $L = B'\,G' = L_2$ we shall have

$$H = \frac{L_1^3 - L_2^3}{3\,\beta^2\,A'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

Cor. (1).—If we suppose the line $H\,I$ to be parallel to the surface of the water, and y to be the distance from the surface of the water to this line. Let W equal the weight of a cubic foot of water; then the pressure on the blade of the screw will be $W\,A'\,y$.

If we suppose the blade of the screw to be moved through the angle θ,

$\therefore\ y_1 + H \sin.\theta$ is the depth of the centre of gravity below the surface of the water.

Or, $W y_1 A' + W \frac{L_1^3 - L_2^3}{3\,\beta^2} \cdot \sin.\theta =$ pressure on the blade of screw.

If the screw be a double-bladed screw, then the centre of gravity of the other blade will have been elevated $H \sin.\,\theta$.

$\therefore\ y_1 - H \sin.\theta$ is the depth of the centre of gravity below the surface of the water.

And $W y_1 A' - W \frac{L_1^3 - L_2^3}{3\,\beta^2} \cdot \sin.\,\theta =$ pressure on the other blade of screw.

$\therefore\ 2\,W y_1\,A' =$ the pressure of water on both blades of the screw.

This pressure is constant, and independent of the position of the screw.

By substituting the value of A', as given in Problem (3), we have

$W y_1 \left\{ h^2 \log.\left(\frac{L_1 + r_1\,\beta}{L_2 + r_2\,\beta}\right) + L_1\,r_1 - L_2\,r_2 \right\} =$ the pressure of the water on both blades of the screw.

Note.—If we substitute the values of β_1, L_1, and L_2, as given in cor. (3), prob. (1), in formula (3), prob. (3), we shall have for the case when $r_1 = 0$

$$A' = \frac{h}{2}\left\{ r \sqrt{\left(\frac{2\,\pi\,r}{p}\right)^2 + 1} + \frac{p}{2\,\pi} \log.\left(\frac{2\,r\,\pi}{p} + \sqrt{\left(\frac{2\,r\,\pi}{p}\right)^2 + 1}\right)\right\} \quad . \ (1).$$

And since $\tan. A = \frac{p}{2\,r\,\pi}$ we shall have

$$A' = \frac{h}{2}\left\{ r \operatorname{cosec.} A + r \tan. A \log.(\cot. A + \operatorname{cosec.} A)\right\}$$

$$= h\,r\left\{\frac{\operatorname{cosec.} A + \tan. A \log.\left(\cot. \frac{A}{2}\right)}{2}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

This formula, which expresses the area of the screw-blade in terms of the length, radius, and angle of screw, is more simple than the one given by Professor Main and Thomas Brown, Esq., (see "Marine Steam Engine, page 267).

By using Simpson's formula to compute the area of the curve fig. (4), we shall have

$$A' = \frac{r\,h}{6}\left\{ 1 + 4\sqrt{\left(\frac{2\,\pi\,r}{2\,p}\right)^2 + 1} + \sqrt{\left(\frac{4\,r\,\pi}{2\,p}\right)^2 + 1}\right\} \text{ nearly} \quad . \quad . \quad . \quad (3).$$

Or,

$$A' = \frac{r\,h}{12}\left\{1 + 4\sqrt{\left(\frac{\pi r}{2p}\right)^2 + 1} + 2\sqrt{\left(\frac{2\pi r}{2p}\right)^2 + 1}\right.$$
$$\left. + 4\sqrt{\left(\frac{3\pi r}{2p}\right)^2 + 1} + \sqrt{\left(\frac{4\pi r}{2p}\right)^2 + 1}\right\} \quad . \quad . \quad . \quad (4).$$

Taking the screw-blade, whose dimensions are $r = 2{\cdot}833$, &c., feet, $p = 8$ feet, and $h = 2{\cdot}5$ feet, which is the screw selected by Professor Main (see Marine Steam Engine, page 265), and by using the first formula, we shall have the correct area, $A' = 11{\cdot}0915$ square feet for each blade; and by using the formula (3), we shall have $A' = 11{\cdot}1246$, a little too great.

The approximation recommended by Professor Main gives $A' = 11{\cdot}4615$ (see Marine Steam Engine, page 266). Formula (2) shows, when the angle of a screw is constant, the area of the blade varies as the rectangle of the length and diameter.

CHAPTER II.

PROBLEM I.

Explanation of the Action of Water on the Screw-blade when immersed in the Water.

In the equations which we have obtained in the preceding problems, we have adopted three rectangular planes; the plane of $x\,y$ we shall always fix at right angles to the motion of the vessel which the blade of the screw is destined to propel; and, consequently, the axis of z will be the axis of the axle which drives the screw round in the water, and the depths from the surface of the water will be measured along the axis of y, the axis of x being horizontal at right angles to the motion of the vessel. In fact the axis of y is vertical, and the axis of z is in the direction of motion, and that of x is at right angles to the former two.*

There are, however, certain states in which if the screw-blade be fixed, the pressure of the water upon the screw-blade will produce no effect to propel the vessel. To point out these states we deem to be an object of considerable importance, and we think it will be best effected in the following manner:

If the screw-blade have no angular velocity, the pressure of the water upon it can produce no effect to propel the vessel. This is obvious.

* In whatever state the screw-blade be placed, the pressure of the water at a point upon it will be in the direction of a normal at that point, or perpendicular to the surface of the screw.

If the screw-blade have an infinite angular velocity, the pressure of the water upon it can produce no effect to drive the vessel on. This is also obvious, for the screw-blade becomes in this case like a cylinder revolving upon its axis.

If the screw-blade advance uniformly the length of its pitch during the time it makes one revolution uniformly, the pressure of the water upon it can produce no effect to propel the vessel; because the edge of the screw-blade, in this case, is always presented to the water, which is cut by the edge of the blade without its surface being pressed unequally by the water surrounding it.

PROBLEM II.

To find the Surface of Vanishing pressure when the Screw works in Still Water.

Let $w =$ angular velocity of the screw at a distance of one foot from the axis.

$\pi = 3{\cdot}141$, &c., &c.

$\therefore 2\pi =$ space described in one revolution of the screw:

$$\therefore 2\pi = wt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1).$$

by the dynamics of uniform motion where t is the time in seconds during one revolution.

Again, let $v =$ velocity of the screw in the direction of its axis.

$p =$ the space passed over during one revolution of the screw.

$$\therefore p = vt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

From (1) and (2) we shall obtain

$$w = \left(\frac{2\pi}{p}\right) \times v \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3).$$

If the relation between the angular velocity and the velocity of translation of the screw expressed in equation (3) obtain, the screw will pass through the water without its surface being pressed in the direction of motion. Hence, when we know the velocity of a vessel, we can readily obtain from (3) the velocity of the screw, when it produces no effect upon the water, but simply presents its edge to the water, and cutting it.

The particular surface traced out by the screw, when the relation (3) obtains, may be called the *surface of vanishing pressure.*

The above relation obtains only when the water through which the screw passes has no motion; when the screw is in the position expressed by formula (3), the surface of the screw-blade will rub on the water, and produce a small amount of friction; but for this, the whole amount of the force of the engine would be absorbed in the friction of its parts and turning the screw round.

PROBLEM III.

To find the Surface of Vanishing Pressure when the Water is moving with a Velocity V in the direction of the Axis of the Screw.

We shall have, as in the last Problem,

$$2\pi = w t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1).$$

But, instead of the screw advancing in the direction of its axis the length of its pitch, as in the last Problem, it must advance beyond that distance a space due to the velocity of the water in one revolution of the screw.

Hence we shall have

$$t V + p = v t$$

from which we obtain

$$t = \frac{p}{v - V} \cdot \quad . \quad . \quad . \quad . \quad . \quad (2).$$

From (1) and (2) we have

$$w = \left(\frac{2\pi}{p}\right)(v - V) \quad . \quad . \quad . \quad (3).$$

This equation gives us the surface of vanishing pressure when the water is moving with a velocity V in the direction of the axis of the screw.

Cor. (1).—If the water moves with a velocity V in an opposite direction to the axis of the screw, then we shall have, by similar reasoning to the above,

$$w = \left(\frac{2\pi}{p}\right)(v + V) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4).$$

PROBLEM IV.

To change the Velocity in Feet per Second to Miles per Hour, and Knots per Hour; and, also, the Angular Velocity to the Number of Revolutions per Minute.

Let v = velocity or feet per second.

$$\therefore \frac{v}{176 \times 10 \;\; 3} = \text{ miles per second.}$$

$$\text{and } \frac{v \times 60 \times 60}{176 \times 10 \times 3} = (m\,h) = \text{ miles per hour.}$$

$$\therefore v = \frac{22}{15}(m\,h) \quad . \quad . \quad . \quad (1).$$

$$= m\,h + \frac{7\,(m\,h)}{15}$$

Hence, in practice, we may add to miles per hour its half, and we shall obtain the velocity in feet per second nearly; this gives a little too much.

$$(m\,h) = \frac{15\,v}{22}$$

$$= \left(\frac{11}{22} + \frac{4}{22}\right)v$$

$$= \frac{v}{2} + \frac{2\,v}{11}$$

Hence, when we know the velocity in feet per second, if we take one-half of it and add this to two-elevenths of it, we shall obtain miles per hour.

Let $a =$ the number of feet in a knot or nautical mile.

Now $v =$ velocity or feet per second.

$$\therefore \frac{v}{a} = \text{knots per second.}$$

$$\frac{v \times 60 \times 60}{a} = K = \text{knots per hour.}$$

$$\therefore v = \frac{a\,K}{3600} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

Again.—Let $n =$ the number of revolutions per minute.

$$\therefore \frac{n}{60} = \text{the number per second.}$$

$2\,\pi \times \frac{n}{60} =$ the space described by a point at one foot from the axis in a second.

$$\therefore w = \frac{\pi}{30} \times n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3).$$

Scholium.—If we substitute the values of v, w, V, in equations (3), (3), and (4), Probs. (2) and (3), we shall obtain

$$\frac{\pi}{30} \times n = \frac{2\,\pi}{p} \times \frac{a\,K}{3600}$$

$$\therefore n = \frac{a\,K}{60\,p} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4).$$

when the water is at rest.

$$\frac{\pi}{30}\,n = \frac{2\,\pi}{p}\left(\frac{a\,K}{3600} - \frac{a\,K'}{3600}\right)$$

$$\therefore n = \frac{a}{60\,p}\,(K - K') \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5).$$

where K' is knots per hour corresponding to V feet per second.

$$n = \frac{a}{60\,p}\left(K + K'\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (6).$$

when the velocity of the water is opposite to the velocity of the screw.

These equations, and the surface of vanishing pressure, which they enable us to determine, will assist us in the explanation of some apparent anomalies which practice has detected.

At page 249, Falconer's Marine Dictionary, modernized and enlarged by W. Burney, L.L.D., master of the Naval Academy, Gosport, it is stated

that a knot or nautical mile is 6120 feet. Substituting this value in equations (4), (5), and (6), we shall have

$$n = \frac{102}{p} K \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7).$$

when the water in which the screw is revolving is at rest.

$$\text{And, } n = \frac{102}{p} \left(K - K' \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8).$$

when the water in which the screw is revolving is moving in the direction of the vessel's motion with a velocity of K' knots per hour.

$$\text{And, } n = \frac{102}{p} \left(K + K' \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9).$$

when the water in which the screw is revolving is moving in the direction opposite to the direction of the vessel's motion, with a velocity of K' knots per hour.

The ratio $\frac{102}{p}$ should be legibly written on the surface of every screw-blade; it is the number which will enable us to determine the angular motion of the screw when it produces no effect upon the water through which it passes: when this circumstance is fully determined we can judge pretty well whether the screw is acting effectively on the water to produce motion in the vessel to which it is attached.

The following table has been obtained from printed reports, and may be relied upon as being as accurate as the peculiar circumstances in which the vessel is placed during the experiments will admit of:

Results of Trials of Screw Steam-Vessels.

Names.	Draught of Water.				Area of Midship Section.	Displacement in Tons.	Revolutions per Minute.		Pressure per in. on the Safety Valve.	Power Exerted.	Speed of Vessel.	Screw Propeller.						n.
	Forward.		Abaft.				Engine.	Screw.				Length.		Diameter.		Pitch.		
	Ft.	in.	ft.	in.	sq ft	No.	No.	No.	lbs.	horse	knots.	ft.	in.	ft.	in.	ft.	in.	
Ajax	20	4½	22	3	702	2819	41	41	—	854	6.43	3	2	16	1	19	6	33
Arrogant	16	10	18	9	526	2257	62	62	—	682	7.5	2	6	15	6	15	0	51
Encounter	11	5½	12	2½	321	1197	77.5	77.5	7	693	10.25	2	8	12	0	15	7	67
Termagant	16	2	17	10	588	2391	32	64	14	1124	8.40	3	0	15	6	18	0	47
Plumper	10	7	11	3	205	540	46	115	6	148	7.42	1	0	8	9	5	7	135
Plumper	10	7	11	8	208	546	54½	136¼	13	1139	7.23	0	10	8	9¼	4	6½	162
Termagant	16	2	18	0	591	2408	36½	73	14	1333	9.51	2	10½	15	6	17	9½	54
Fairy	5	1	7	1	104	251	43	215	—	—	12.18	1	0	6	6	8	0	155

The value of n, in column n, is computed from formula (7), giving the number of revolutions per minute made by the screw when it works on the surface of vanishing pressure.

If we direct our attention to the column n computed by the formula, and the number of revolutions actually made by the screw per minute, we shall observe some curious anomalies.

In the case of the *Ajax* we observe the number of revolutions actually made by the screw is 41, and the number of revolutions to be made when the screw is on the surface of vanishing pressure only 33. Hence we can see there is a pressure upon the screw propelling the vessel along. The same remark will apply to all the other vessels except the *Plumper*, which presents a curious anomaly in two cases, when propelled by screws of different pitches.

We see in the case of the *Plumper*, the number of revolutions actually made by the screw is 115, and the number of revolutions to be made by the screw before it is working on the surface of vanishing pressure is 135, which is greater than the former. This circumstance shows that the *Plumper* passes over a space greater than the pitch of the screw during one revolution.

Before we attempt to give an explanation to this anomaly, which has been perplexing to practical men, it will be necessary to make the following remarks:

When the vessel advances the length of the pitch of the screw during the time it makes one revolution, the vessel is said to be moving as fast as the screw.

When the vessel advances a distance less than the pitch of the screw during the time it makes one revolution, the vessel is said to move slower than the screw; and the distance, which is the difference between the pitch of the screw and the space through which the vessel is propelled during one revolution of the screw, is called by practical men the *slip* of the screw.

When the vessel advances a distance greater than the pitch of the screw, the vessel is said to move faster than the screw. With these conventions I have nothing to do but explain them, as they are used and known by practical men; but I cannot refrain from thinking they are unhappily selected to explain the phenomena of the action of the screw on the water to propel the vessel along.

To find S the slip of the screw, when the vessel is moving with a velocity v, and the screw moving with an angular velocity w, we have

$$S = p - vt$$

$$\text{but } 2\pi = wt$$

$$\therefore S = p - \frac{2\pi v}{u} = \frac{wp - 2\pi v}{w}$$

By Problem 4 we have $u = \frac{\pi}{30} n$; and $v = \frac{51}{30} K$

$$\therefore S = p - 102 \frac{K}{n}$$

If $p > 102 \frac{K}{n}$ the vessel has a slip, and is not moving so fast as the screw.

If $p < 102 \frac{K}{n}$ the vessel is moving faster than the screw.

If $p = 102 \frac{K}{n}$ the slip is nothing; showing that the vessel is moving as fast as the screw, and if the screw be moving in this case in still water, there will be no pressure upon it to be effective in propelling the vessel along. The screw, in this case, is moving on the surface of vanishing pressure.

When the screw is working nearly on the surface of vanishing pressure, the motion of the vessel will be retarded until the screw is working at a greater distance from this surface, when the motion of the vessel will be accelerated. Hence there is a succession of alternate accelerations and retardations, until the resistance of the vessel, and the power of the engine, through the medium of the screw, are in equilibrium, when there will be a uniform motion in both the vessel and the screw. This phenomenon was observed in the *Fairy*, when she was moving at half-speed, by a writer in the Practical Mechanic and Engineer's Magazine, vol. i., second series, page 296, who signs "Go-Ahead:" "The screw appears to slip and hold alternately, instead of the slip being, as in the paddle-wheel, a constant steady quantity; that is, the screw appears to hold, and propel the vessel, for two revolutions, and then suddenly to slip one, when it is again brought up for two revolutions as before." This fact is accounted for by such a relation subsisting between the velocity of the vessel and the rotation of the screw, as is required for the screw to work on the surface of vanishing pressure. This being the case, the speed of the vessel must diminish, although the angular motion of the screw be the same, until the screw is working at a distance from the surface of vanishing pressure, and then the motion of the vessel will be increased, &c., &c.

The situation of the screw used to propel the vessel is fixed in the deadwood of the run, near the stern-post, and the axis is so situated that the whole of the screw is immersed in the water. Those who are unacquainted with the situation of the screw-propeller, may see with advantage Professor Woodcroft's Steam Navigation, page 120, where an interesting description of his valuable invention of the variable pitch is given. I shall have an occasion to refer to the subject of this invention more extensively in a subsequent page.

When a vessel is moving through the water, the bow and the fore part of the vessel to the greatest section displace the water, while the after part from the midship section to the stern leaves the water as the vessel proceeds. The consequence of this is that the pressure of water against a part of the vessel before the greatest section is greater when the vessel is in motion than when it is at rest, and the pressure of the water on a part abaft the greatest section is less when the vessel is in motion than when it is at rest. Hence the fore part of the vessel will be raised, and the after part lowered, when it is moving rapidly through the water: therefore, one section of the vessel will not be raised or lowered by this circumstance.

This section we shall call the *neutral section.* It will be readily seen, as the ship or vessel advances, that the water pressing on the after part of it flows into the void made by the successive positions which the vessel is compelled to take. Therefore, there will be produced in the wake of the vessel a current, the velocity of which varies from the velocity of the vessel at a point adjacent to the vessel, to nothing at a point in the wake of the vessel: this point is indefinitely fixed, and its distance from the stern of the vessel will depend upon its velocity, and on the form of its after part. The velocity of the water in the wake of the vessel diminishes rapidly as the distance from the stern of the vessel increases; and at no great distance from the stern, the velocity ceases altogether.

In a paper on Physical Data, printed in the Manchester Literary and Philosophical Society's Memoirs, I ventured to call the water disturbed by a vessel passing uniformly through it, the *solid of disturbance;* therefore, the point in the wake of the vessel, at which the velocity of the current becomes nothing, is the intersection of the axis of the vessel with the *solid of disturbance.*

Hence the surface of the *solid of disturbance* will trace out the points in the wake of the vessel, at which the velocity of the water vanishes.

If the screw is so situated, when the vessel is moving, as to be within the limits of the *solid of disturbance,* then the screw is not working in still water, but in water which moves in the direction of the vessel's motion.

In this case the *surface of vanishing pressure* is not in the same position as it would be if the water, in which the screw works, were at rest; but it is determined in conformity with formula (8), scholium to prob. (4).

In this formula we shall require the velocity of the water, in the direction of the vessel's motion, in which the screw is working, before we can compute the number of revolutions to be made by the screw, when it is working on the *surface of vanishing pressure;* and it is evident, from formula (8), that the number of revolutions will be less as the velocity of the water in the direction of the vessel's motion is greater. In the case of the *Plumper,* the velocity of the water in which the screw works is considerable; and, therefore, the number of revolutions of the screw, when it is working on the *surface of vanishing pressure,* is less than it would be if the water were at rest. This is the reason why vessels are sometimes found to be moving faster than the screw, a circumstance which has frequently puzzled practical men. This explanation of an apparent paradox in the screw question, is offered with great respect to the opinions of practical men. In a careful consideration of this subject it has appeared to me satisfactory; as it assigned a reasonable cause for a fact which had been previously regarded as an anomaly.

From this it appears to be inferable, that the position of the screw must be as close to the vessel as circumstances will permit, or the screw must be

placed in the position in which the velocity of the water in the direction of the vessel's motion is the greatest.

In the *Plumper* this condition is complied with to a greater extent than in many other vessels.

With respect to the determination of the *solid of disturbance,* and to the velocity of the water in the wake of the vessel, experiments which are on record avail but little to the development of these important questions in naval architecture.

PROBLEM V.

To find the Moment of Inertia of Smith's Screw-blade.

It may not be entirely out of place here to make a statement of a theorem in the calculus of the moments of inertia, which is of considerable importance when the figure or body is divided into several parts.

Let $\Sigma\, m_1\, r_1^2$ = one group of moments, and K_1 their radius of gyration;
$\Sigma\, m_2\, r_2^2$ = ditto K_2 ditto;
$\Sigma\, m_3\, r_3^2$ = ditto K_3 ditto;
&c., &c., &c.

Then we shall have

$$\Sigma\,(m_1) \times K_1^2 = \Sigma\, m_1\, r_1^2$$
$$\Sigma\,(m_2) \times K_2^2 = \Sigma\, m_2\, r_2^2$$
$$\Sigma\,(m_3) \times K_3^2 = \Sigma\, m\;\; r_3^2$$

If K = the radius of gyration of the whole mass, we shall have

$$\{\Sigma\,(m_1) + \Sigma(m_2) + \&c.\}\, K^2 = \Sigma\, m_1\, r_1^2 + \Sigma\, m_2\, r_2^2 + \&c.$$
$$= \Sigma\,(m_1) \times K_1^2 + \Sigma\,(m_2) \times K_2^2 + \&c. \quad (1).$$

Now, if the groups $\Sigma\,(m_1)$, $\Sigma\,(m_2)$, &c., be equal, and similarly placed, and there be n of them, we shall have

$$n\,\Sigma\,(m_1) \times K^2 = n\,\Sigma\,(m_1)\, K_1^2$$

Hence $K = K_1$

Therefore $n\,\Sigma\,(m_1) \times K^2 = n\,\Sigma\, m_1\, r_1^2$ . . . . . . (2).

$$\text{And } K^2 = \frac{\Sigma\, m_1\, r_1^2}{\Sigma\,(m_1)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3).$$

Every point in the helix situated at x distance from the axis of the screw, is the same distance x from the axis of the screw.

Put K = radius of gyration of the blade;
A' = area of blade.

$$\therefore A'\, K^2 = \int_0^r x^2 \sqrt{\beta^2 x^2 + h^2}\,.\, dx$$

$$= \beta^2 \int_0^r \frac{x^4\, d\,x}{\sqrt{\beta^2 x^2 + h^2}} + h^2 \int_0^r \frac{x^2\, d\,x}{\sqrt{\beta^2 x^2 + h^2}}$$

$$= \left(\frac{r^3}{4} + \frac{h^2\, r}{8\, \beta^2}\right) \sqrt{\beta^2 r^2 + h^2} + \frac{h^4}{8\, \beta^3} \log. \left(\frac{h}{\beta\, r + \sqrt{\beta^2 r^2 + h^2}}\right)$$

See Professor Young's Integral Calculus, page 51.

Hence, the moment of inertia of a double-bladed screw will be

$$2\, A'\, K^2 = \left(\frac{r^3}{2} + \frac{h^2\, r}{4\, \beta^2}\right) \sqrt{\beta^2 r^2 + h^2} + \frac{h^4}{4\, \beta^3} \log. \left(\frac{h}{\beta\, r + \sqrt{\beta^2 r^2 + h^2}}\right) \quad . \quad . \quad (4).$$

Cor. (1).—Formula (4) may be expressed in terms of the length, pitch, and diameter of the screw, by means of the formula $\beta = \frac{2\, h\, \pi}{p}$; substitute this value in equation (4), and we shall have

$$2\, A'\, K^2 = \frac{h}{2} \left\{ \left(r^3 + \frac{r\, p^2}{8\, \pi^2}\right) \sqrt{\frac{4\, r^2\, \pi^2}{p^2} + 1} - \frac{p^3}{16\, \pi^3} \log. \left(\frac{2\, r\, \pi}{p} + \sqrt{\frac{4\, r^2\, \pi^2}{p^2} + 1}\right) \right\} (5).$$

Cor. (2).—This formula may be expressed in terms of the angle of the screw, by the formula, tan. $A = \frac{p}{2\, r\, \pi}$; substitute this value in (5), and we shall have

$$2\, A'\, K^2 = \frac{h\, r^3}{2} \left\{ \left(1 + \frac{\tan.^2 A}{2}\right) \operatorname{cosec.} A - \frac{\tan.^3 A}{2} \log. \left(\cot. \frac{A}{2}\right) \right\} \quad . \quad . \quad . \quad . \quad (6).$$

The hyperbolic logarithms are to be used in each of the above formulæ. When the angle of a screw is constant, the moment of inertia is directly as the length and the cube of the diameter.

Further, when the length and the angle are constant, the moment of inertia varies directly as the cube of the diameter. Likewise, when the diameter and the angle are constant, the moment of inertia is directly proportional to the length.

Cor. (3).—According to note, page 9, twice the area of the blade is

$$2\, A' = h \left\{ r \sqrt{\frac{4\, r^2\, \pi^2}{p^2} + 1} + \frac{p}{2\, \pi} \log. \left(\frac{2\, r\, \pi}{p} + \sqrt{\frac{4\, r^2\, \pi^2}{p} + 1}\right) \right\}$$

$$\therefore K^2 = \frac{h\, (4\, r^3\, \pi^2 + r\, p^2) \sqrt{\frac{4\, r^2\, \pi^2}{p^2} + 1} - A'\, p^2}{16\, \pi^2\, A'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7).$$

Cor. (4).—If the blade of the screw has a thicknes T, the radius of gyration is not altered, but the moment of inertia becomes

$2\, A'\, T\, K^2$ instead of $2\, A'\, K^2$, as in the above formula.

And if S be the weight of a unit of the metal of which the screw is made, then $2\, A'\, T = \frac{W}{S}$, where W is the weight of the screw, and $\frac{W}{S}\, K^2$ = moment of inertia.

PROBLEM VI.

Given the Angular Velocity of the Screw, to find the Normal Velocity.

By referring to Cor. (1), Prob. (2), Chap. I., we shall observe that the blade of the screw is formed of an infinite number of concentric helices, having the same pitch, but a varying angle.

Let the figure $B\,B\,D$ be the development of the helix, at a distance x from the axis of the screw.

$B\,G$ is the length of the helix.

Put $w =$ angular velocity of any point in the helix, at a distance unity from the axis.

$\therefore x\,w =$ angular velocity of every point in $B\,G$.

Let $B\,G$ move with a uniform velocity $x\,w$, into the parallel position $K H'$. $G H' K B$ is evidently a parallelogram, whose opposite sides and angles are equal. Draw $G I$ perpendicular to $B\,G$ or $K H'$. Then, during the time the point G moves to H', the helix $B\,G$ moves over $G I$, parallel to itself.

Put $V =$ normal velocity, or velocity in $G I$.

$$\angle\, G B K = \angle\, G H' I = \theta.$$

Now, in uniform motion, the velocities are proportional to the spaces passed over.

$$\therefore \frac{x\,w}{V} = \frac{G H'}{G I} = \frac{G H'}{G H'.\sin.\theta} = \frac{1}{\sin.\theta}$$

$$\therefore V = x\,w\sin.\theta \quad . \quad . \quad . \quad . \quad . \quad (1).$$

Cor. (1).—If we suppose the screw to advance the distance $K L$, in the direction of its axis, during the time it would move uniformly over the space $G H'$; then $L N$ will be the position of the screw, instead of $K H'$, as in the former case: draw $L M$ perpendicular to $K H$.

Put $v =$ velocity in the direction of the axis of the screw.

$$\therefore \frac{v}{V'} = \frac{L K}{L M} = \frac{1}{\cos.\theta} \quad \therefore V' = v\cos.\theta \quad . \quad (2).$$

$$\therefore V - V' = x\,w\sin.\theta - v\cos.\theta \quad . \quad . \quad (3).$$

which is the normal velocity in this case.

Cor. (1).—Since $\tan.\theta = \frac{h}{\beta\,x}$, we have

$$\sin.\theta = \frac{h}{\sqrt{\beta^2 x^2 + h^2}}\,;\ \text{and}\ \cos.\theta = \frac{\beta\,x}{\sqrt{\beta^2 x^2 + h^2}}.$$

Substitute these values in equations (1) and (3), and we shall have

$$V = \frac{h\,x\,w}{\sqrt{\beta^2 x^2 + h^2}} \quad . \quad . \quad (4).$$

$$\text{And } (V - V') = \frac{(h\,w - \beta\,v)\,x}{\sqrt{\beta^2 x^2 + h^2}} \quad . \quad . \quad (5).$$

These equations determine the normal velocity in the two cases.

PROBLEM VII.

To find the Forces acting upon the Screw-blade, when the Vessel is at Rest, and the Screw moving with an Angular Velocity w.

Let P be the moment of the engine applied to the screw.

$\therefore \frac{P}{2 A' T K}$ = accelerating force of P on the screw . . (1).

Besides this accelerating force there is another called resistance, depending upon the normal velocity, and the density, &c., of the water.

Let $n \, \mathrm{f} \, (V)$ = accelerating force of resistance perpendicular to the blade of the screw, where n is a constant to be determined by experiment.

This accelerating force may be represented in magnitude and direction by the line $G I$, which may be decomposed into two others $G P$, $P I$, perpendicular and parallel to $G H'$.

Hence $G P = G I \times \cos. \theta$, and $P I = G I \times \sin. \theta$.

Therefore $n \, \mathrm{f} \, (V) \cos. \theta$ = accelerating force in the direction of the axis of the screw.

and $n \, \mathrm{f} \, (V) \sin. \theta$ = accelerating force perpendicular to the axis of the screw.

The force $n \, \mathrm{f} \, (V) \cos. \theta$ is expended by pressing upon the blade of the screw, and if the vessel were at liberty to move, this force would move it.

The force $n \, \mathrm{f} \, (V) \sin. \theta$ resists the motion of rotation at any point x distance from the axis.

Therefore $\frac{n \, \mathrm{f} \, (V) \, h \, x}{\sqrt{\beta^2 x^2 + h^2}}$ = moment of accelerating force of resistance at a point x distance from the axis.

And since all the points in the helix, whose length is $\sqrt{\beta^2 x^2 + h^2}$, are situated at the same distance x from the axis of the screw, we shall have

$F = 2 \, n \, h \int_0^r \mathrm{f} \, (V) \, x \, d \, x$ = moment of accelerationg force of both blades of the screw . . . . (2).

Therefore $\frac{P}{2 A' T K^2} - 2 n h \int_0^r f(V) \, x \, d x$ = accelerating force to produce angular motion in the screw . . . . (3).

Cor. (1).—If we suppose the resistance to be proportional to the square of the normal velocity, which is the common theory of resistances,

$\therefore \mathrm{f} \, (V) = \frac{h^2 w^2 x^2}{\beta^2 x^2 + h^2}$, and equation (3) becomes

$$\frac{P}{2 A' T K^2} - 2 \, n \, h^3 \, w^2 \int_0^r \frac{x^3 \, d \, x}{\beta^2 x^2 + h^2} = \frac{P}{2 A' T K^2} - \frac{n \, h^3 \, w^2}{\beta^4} \left(r^2 \beta^2 + h^2 \log \frac{h^2}{\beta^2 r^2 + h^2} \right)$$

= accelerating force to produce angular motion . . (4).

See note 1, chap. ii.

PROBLEM VIII.

To find the Forces acting on the Blade of the Screw when the Vessel is moving with a velocity v, and the Screw moving with an angular velocity w.

Reasoning as in the last problem, we shall have

$\frac{P}{2\,A'\,T\,K^2}$ = accelerating force of P on the screw . . . (1).

$\therefore\ n\ \mathrm{f}\,(V - V')\cos.\,\theta$ = accelerating force in the direction of the axis of the screw.

And $n\ \mathrm{f}\,(V - V')\sin.\,\theta$ = accelerating force perpendicular to the axis of the screw.

$\therefore\ F' = 2\,n\,h\int_0^r \mathrm{f}\,(V - V')\,x\,d\,x$ = moment of accelerating force of both blades of the screw.

Therefore $\frac{P}{2\,A'\,T\,K^2} - 2\,n\,h\int_0^r \mathrm{f}\,(V - V')\,x\,d\,x$ = accelerating force to produce angular motion . . . . (2).

If we suppose $m\,A''\,v^2$ to represent the accelerating force that resists the motion of the vessel, which is not far from the truth in ordinary velocities, where A'' is the area of the midship section, and $m\,A''$ the accelerating force with a velocity unity;

then $2\,.\,\beta\ n\int_0^r \mathrm{f}\,(V - V')\,x\,dx - m\,A''\,v^2$ = accelerating force of both blades to produce motion in the vessel . . . . . . (3).

See note 2, at the end of chap. ii.

Cor. (1).—Taking the resistance to be proportional to the square of the normal velocity, we shall have

$$\int_0^r f\,(V - V')\,x\,dx = (h\,w - \beta\,v)^2\int_0^r \frac{x^3\,d\,x}{\beta^2\,x^2 + h^2}$$

$$= \frac{(h\,w - \beta\,v)^2}{2\,\beta^4}\left(r^2\,\beta^2 + h^2\log.\frac{h^2}{r^2\,\beta^2 + h^2}\right)$$

Substitute this value in (2) and (3), we shall have

$\frac{P}{2\,A'\,T\,K^2} - \frac{n\,h\,(h\,w - \beta\,v)^2}{\beta^4}\left(r^2\,\beta^2 + h^2\log.\frac{h^2}{r^2\,\beta^2 + h^2}\right)$ = accelerating force to produce angular motion . . . . (4).

$\frac{n\,(h\,w - \beta\,v)^2}{\beta^3}\left(r^2\,\beta^2 + h^2\log.\frac{h^2}{r^2\,\beta^2 + h^2}\right) - m\,A''\,v^2$ = accelerating force to produce motion in the ship . . . . (5).

When $h w - \beta v = 0$, there are no resistances acting on the surface of the screw-blade, or the screw is working on the *surface of vanishing pressure*.

$$\frac{w}{v} = \frac{\beta}{h} = \frac{1}{r \tan. A} = \left(\frac{2\pi}{p}\right);$$

which is the same result arrived at by Prob. (2).

PROBLEM IX.

To find the Velocity of the Screw when the Vessel is at rest.

By dynamics we have $\frac{d w}{d t}$ = angular accelerating force of the screw.—See Dr. Whewell's Dynamics, Part II., pp. 185, 118.

$$\therefore \frac{d w}{d t} = \frac{P}{2 A' T K^2} - \frac{n h^3 w^2}{\beta^4}\left(r^2 \beta^2 + h^2 \log. \frac{h^2}{r^2 \beta^2 + h^2}\right)$$

$$= a^2 - b^2 w^2$$

$$\therefore d t = \frac{d w}{a^2 b^2 w^2} = \frac{1}{2 a}\left\{\frac{d w}{a + b w} + \frac{d w}{a - b w}\right\}$$

$\therefore t = \frac{1}{2 a b} \log. \left(\frac{a + b w}{a - b w}\right)$: solve this equation with respect to w, and we have

$w = \frac{a}{b} - \frac{2 a}{b(1 + E^{2 a b t})}$ the angular velocity at any time t.

When t is considerable, the last term of the above expression will be small, and the screw will then turn round with a uniform velocity $\frac{a}{b}$, called the *terminal* velocity.

This result could be more readily obtained, by supposing the terminal velocity to be greatest which is obtained when the accelerating force is nothing—

$$\therefore a^2 - b^2 w^2 = 0, \text{ or } w = \frac{a}{b}$$

PROBLEM X.

To find the Velocity of the Screw when the Vessel is in motion.

It will be readily seen by the last problem, that

$$\frac{d w}{d t} = a^2 - \frac{b^2 (h w - \beta v)^2}{h^2} \quad . \quad . \quad . \quad (1).$$

$$\frac{d v}{d t} = \frac{\beta b^2 (h w - \beta v)^2}{h^3} - m A'' v^2 \quad . \quad . \quad (2);$$

where $a^2 = \frac{P}{2 A' T K^2}$, and $b^2 = \frac{n h^3}{\beta^4}\left(r^2 \beta^2 + h^2 \log. \frac{h^2}{r^2 \beta^2 + h^2}\right)$

From (1) and (2) we obtain

$$\frac{d\,w}{d\,v} = \frac{h\left\{a^2 h^2 - b^2 (h\,w - \beta\,v)^2\right\}}{\beta\,b^2 (h\,w - \beta\,v)^2 - m\,h^3 A''\,v^2} \quad . \quad (3).$$

From this equation w may be found in terms of v; and then equations (1) and (2) may be integrated.

But since the terminal velocity is obtained when the accelerating force is zero, and the terminal velocity is the one to which the vessel and screw approach as their uniform velocity as the time increases, we shall have from (1) and (2)

$$a^2 h^2 = b^2 (h\,w - \beta\,v)^2 \quad . \quad . \quad . \quad . \quad . \quad (4).$$

$$\beta\,b^2 (h\,w - \beta\,v)^2 = h^3\,m\,A''\,v^2 \quad . \quad . \quad . \quad . \quad (5).$$

From these two equations we have

$$v^2 = \frac{\beta\,a}{m\,A''\,h} = \frac{P\,\beta}{2\,m\,A''\,T\,A'\,K^2\,h} \quad . \quad . \quad (6).$$

$$w = \frac{a}{b} + \frac{a\sqrt{\beta^3}}{\sqrt{m\,A''\,h^3}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7).$$

By substituting the value of $A'\,K^2$, in formula (6), problem (5), we have

$$v^2 = \frac{2\,P\,\text{cot.}\,A}{m\,A''\,T\,h\,r^4\left\{\left(1 + \frac{\text{tan.}^2 A}{2}\right)\text{cosec.}\,A - \frac{\text{tan.}^3 A}{2}\cdot\text{log.}\left(\text{cot.}\frac{A}{2}\right)\right\}} \quad . \quad (8).$$

From formula (8) it will be readily inferred, when the angle of the screw is constant, that the velocity is inversely proportional to the square root of the length, and to the square of the diameter.

Hence, by diminishing $r^2\sqrt{h}$ the velocity of the vessel will be increased.

By taking equations (6) and (7) we shall have

$$v = a\,\frac{\sqrt{\beta}}{\sqrt{m\,A''\,h}}$$

$$w = a\left\{\frac{b\sqrt{\beta^3} + \sqrt{m\,A''\,h^3}}{b\sqrt{m\,A''\,h^3}}\right\}$$

$$\therefore \frac{v}{w} = \frac{b\,h\sqrt{\beta}}{b\sqrt{\beta^3} + \sqrt{m\,A''\,h^3}} \quad . \quad . \quad . \quad (9).$$

Substitute this value in the equation, expressing the slip of the screw.—See scholium to prob. (4), chap. (2).

$$\therefore s = p - \frac{2\,\pi\,b\,h\sqrt{\beta}}{b\sqrt{\beta^3} + \sqrt{m\,A''\,h^3}} \quad . \quad . \quad (10).$$

Hence the slip of the screw is independent of the power applied, and of the moment of inertia of the screw. The slip depends therefore upon the dimensions of the screw, the resistance of the vessel at a unit of velo-

city, and the resistance perpendicular to the surface of the screw at a unit of velocity.

Since $b^2 = \frac{n\,h^3}{\beta^4}\left\{r^2\,\beta^2 + h^2 \log.\frac{h^2}{r^2\,\beta^2 + h^2}\right\}$

$$= \frac{n\,h^5}{\beta^4}\left\{\frac{r^2\,\beta^2}{h^2} + \log.\left(\frac{1}{\frac{r^2\,\beta^2}{h^2} + 1}\right)\right\}$$

$$= n\,h\,r^4 \tan.^4 A\left\{\cot.^2 A + 2\log.(\sin. A)\right\}; \text{ since } \frac{r\,\beta}{h} = \cot. A$$

$$\therefore\ b = \sqrt{n}\,.\,\sqrt{h}\,.\,r^2 \tan.^2 A\ \sqrt{\cot.^2 A + 2\log.(\sin. A)}$$

And $s = p - \frac{2\,\pi\,.\,\sqrt{n}\,.\,\sqrt{h}\,.\,r^3 \tan.^3 A\left\{\cot.^2 A + 2\log.(\sin. A)\right\}^{\frac{1}{2}}}{b + \sqrt{m\,A''}\,r^3 \tan.^3 A}$. (11).

Hence, when the slip is the least, $\frac{\sqrt{h}\,.\,r^3 \tan.^2 A\left\{1 + 2\tan.^2 A \log.(\sin. A)\right\}^{\frac{1}{2}}}{b + \sqrt{m\,A''}\,r^3 \tan.^3 A}$ is a maximum when the pitch is constant.

From equation (4) we have

$$h\,w - \beta\,v = \frac{a\,h}{b}$$

$$\therefore \frac{h\,w}{v} = \frac{a\,h}{b\,v} + \beta = \frac{a\,h + b\,\beta\,v}{b\,v}$$

$$\therefore \frac{v}{w} = \frac{h\,b\,v}{a\,h + b\,\beta\,v}$$

$$\therefore\ s = p - \frac{2\,\pi\,h\,b\,v}{a\,h + b\,\beta\,v} \text{ from which we obtain}$$

$$p - s = \frac{2\,\pi\,h\,b\,v}{a\,h + b\,\beta\,v}$$

$$\therefore \frac{1}{p - s} = \frac{a\,h + b\,\beta\,v}{2\,\pi\,h\,b\,v} = \frac{a}{2\,\pi\,b\,v} + \frac{\beta}{2\,\pi\,h}$$

$$\left.\begin{aligned} \therefore\ v &= \frac{a\,h}{b} \times \frac{p - s}{2\,\pi\,h - \beta\,(p - s)} \\ \text{and } w &= \frac{a\,h}{b} \times \frac{2\,\pi}{2\,\pi\,h - \beta\,(p - s)} \end{aligned}\right\} \quad . \quad . \quad . \quad (12).$$

When the slip is equal to the pitch, the velocity of the vessel is nothing; the velocity is a maximum when the denominator of the above fraction is equal to nothing, or

$$2\,\pi\,h = (p - s)$$

$$\therefore\ s = p - \frac{2\,\pi\,h}{\beta} = p - 2\,\pi\,r \tan. A = p - p = 0$$

Hence, the velocity of the vessel is the greatest when the slip is nothing.

When the angle and radius of a screw are constant, the slip is less as the length increases up to a point determined by

$$p = \frac{\pi\,\sqrt{n}\,.\,r^3 \tan.^2 A\left\{1 + 2\tan.^2 A \log.(\sin. A)\right\}^{\frac{1}{2}}}{b + \sqrt{m\,A''}\,r^3 \tan.^3 A} \times \sqrt{h}$$

then the slip is nothing.

PROBLEM XI.

To find the angle of the Screw when the v is a maximum; the power applied, radius of Screw and length being constant.

By last problem we have

$v^2 = \frac{P\beta}{2mA'' T h A' K^2}$; where all the elements are constant except $\frac{\beta}{A' K^2}$, which must be a maximum.

$\therefore \frac{A' K^2}{\beta}$ must be a minimum; but $r\ \beta$ tan. $A = h$ $\therefore A' K^2$. tan. A must be a minimum.

Since $A' K^2 = \frac{h r^3}{8}\left\{(2 + \tan.^2 A) \operatorname{cosec.} A - \tan.^3 A \log. \left(\cot. \frac{A}{2}\right)\right\}$ by problem (5), chap. (2). Then we have

$U = (2 \tan. A + \tan.^3 A) \operatorname{cosec.} A - \tan.^4 A. \log. \left(\cot. \frac{A}{2}\right)$ to be made a minimum.

$$\therefore \frac{d\ U}{d\ A} = (2 \sec.^2 A + 3 \tan.^2 A \sec.^2 A) \operatorname{cosec.} A - (2 + \tan.^2 A) \operatorname{cosec.} A$$

$$- 4 \tan.^3 A \sec.^2 A \log. \left(\cot. \frac{A}{2}\right) + \frac{\tan.^4 A. \operatorname{cosec.}^2 \frac{A}{2}}{2 \cot. \frac{A}{2}} = 0$$

$$\therefore (2 + 3 \tan.^2 A) \operatorname{cosec.} A - (2 + \tan.^2 A) \frac{\operatorname{cosec.} A}{\sec.^2 A} - 4 \tan.^3 A \log. \cot. \frac{A}{2}$$

$$+ \frac{\tan.^4 A. \operatorname{cosec.}^2 \frac{A}{2}}{2 \sec.^2 A. \cot. \frac{A}{2}} = 0$$

$$\text{Or, } (2 + 3 \tan.^2 A) \frac{\operatorname{cosec.} A}{\tan.^3 A} - (2 + \tan.^2 A) \frac{\operatorname{cosec.} A}{\sec.^2 A. \tan.^3 A} - 4 \log. \left(\cot. \frac{A}{2}\right)$$

$$+ \frac{\tan. A. \operatorname{cosec.}^2 \frac{A}{2}}{2 \sec.^2 A. \cot. \frac{A}{2}} = 0$$

$$\text{Or, } \frac{2 \cos.^3 A}{\sin.^4 A} + \frac{3 \cos. A}{\sin.^2 A} - \frac{2 \cos.^5 A}{\sin.^4 A} - \frac{\cos.^3 A}{\sin.^2 A} + \cos. A - 4 \log. \cot. \frac{A}{2} = 0$$

$$\therefore \frac{\cos.^3 A}{\sin.^2 A} + \frac{3 \cos. A}{\sin.^2 A} + \cos. A - 4 \log. \left(\cot. \frac{A}{2}\right) = 0$$

$$\therefore \cos. A - \sin.^2 A \log. \left(\cot. \frac{A}{2}\right) = 0$$

$$\therefore \frac{d\,U}{d\,A} = \cot. A - \sin. A . \log. \left(\cot. \frac{A}{2}\right) = 0 \quad . \quad . \quad . \quad . \quad (1).$$

The value $A = \frac{\pi}{2}$ will satisfy this equation.

To discover whether this value gives a maximum or a minimum, we must substitute it in the value $\frac{d^2\,U}{d\,A^2}$

This, being a minus quantity, shows that $A = \frac{\pi}{2}$ cannot be a minimum.

Scholium.—Since U is independent of the radius, length and power applied, we may infer *that the best angle for one screw will be the best for any other.*

The above investigation of U, a minimum, leads to the conclusion that U diminishes with A, and is, therefore, a minimum when $A = 0$.

When A is very small, a great number of revolutions of the screw will be necessary to obtain the same length of the screw; hence, as the angle is diminished, the surface of the blade is increased, and when the angle is nothing, the number of revolutions, as well as the surface, is infinite. This limit cannot be used in practice, in consequence of the great number of revolutions of the screw rendering the distance between each revolution, or the pitch, so small, that the pressure of the water on the surface of the screw-blade in one revolution would be destroyed by the reaction of the surface of the next revolution. To avoid this circumstance, the pitch must not be less than twice the length; this, then, is the limit to which the best angle of the screw approaches.

There is another cause which would render very small angles impracticable; it is the great velocity which would be necessary to drive the screw round, thereby developing a large amount of friction in the engine.

From neglecting to take friction into account, this deduction does not agree very well with experiment, as will be seen from the valuable experiments made by John Fincham, Esq., of Her Majesty's Dock-yard, Portsmouth.

When the angle of the screw is constant we have, by last problem, the square of the velocity, varying inversely as the length and the fourth power of the radius.

$\therefore$ v is a maximum when $\frac{1}{h\, r^4}$ is a max.

If r be constant, v is a max., when $\frac{1}{h}$ is a max.

Hence, when the radius and pitch are constant, the velocity will increase as the length diminishes.

This property is illustrated by the experiments on Her Majesty's ship *Rattler.*—See the "Artizan," 1845, page 55.

	Ft. in.		ft.			
Pitch 11 feet	4 3	length	9	diameter	velocity	10·088
„ 11 „	3 0	„	9	„	„	11·409
„ 11 „	3 0	„	10	„	„	10·78
„ 11 „	2 0	„	10	„	„	10·91

There is a limit to the diminution of the length, in consequence of the increased rotatory motion of the screw developing a large amount of friction in the engine, which is not taken into account in these computations.

In a similar way, the velocity will increase as the diameter decreases, when the angle and the length are constant.

PROBLEM XII.

To find the Velocity of the Vessel when the Pitch is double the length.

$$\therefore \tan A = \frac{h}{r\pi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1).$$

By problem (10), equation (8), we have

$$v^2 = \frac{2 P \cot. A}{m A'' T h r^4 \left\{ \left(1 + \frac{\tan.^2 A}{2}\right) \operatorname{cosec.} A - \frac{\tan.^3 A}{2} \log. \left(\cot. \frac{A}{2}\right) \right\}} \quad . \quad . \quad (2).$$

From (1) we have $h = r \pi \tan. A$. Substitute this in (2), we have

$$v^2 = \frac{4 P}{m A'' T \pi r^5 \tan.^2 A \left\{ (2 + \tan^2 A) \operatorname{cosec.} A - \tan.^3 A \log. \left(\cot. \frac{A}{2}\right) \right\}} \quad . \quad (3)$$

Hence, when the angle is constant, the square of the velocity varies inversely as the fifth power of the radius. Therefore, a large radius will diminish the velocity of the vessel very rapidly. In the foregoing investigations we have supposed the force applied to be constant, for all values of the angular velocities of the screw, and the velocities of translation of the vessel; but a little consideration will show that this is far from being true in practice. When the angular motion is large, a considerable amount of the force applied is deducted from the effective force on the screw, in consequence of the friction of the parts of the machine, which friction becomes larger as the angular motion increases.

It is stated, in Professor Moseley's Engineering and Architecture, page 139, "that the friction of motion is wholly independent of the velocity of motion;" and Professor Moseley remarks, that "this result, of so much importance in the theory of machines, is fully established by the experiments of Morin," "which were made at the expense of the French Government."

From the co-efficients of friction, determined by Morin, the amount of friction in a steam-engine would not be impossible to compute; from this the modulus of the engine would be determined. For an exposition of the modulus of machines see Professor Moseley's Engineering and Architecture, page 162.

By assuming the above law of friction, we shall have as follows:

Put $f =$ moment of the co-efficient of friction, when the velocity is unity.

$\therefore f w =$ the moment of friction when the velocity is w.

For, by the law of Morin, f is constant for all velocities.

Hence this moment of friction must be deducted from P in the foregoing equations.

PROBLEM XIII.

To find the Velocity of the Vessel on the above hypothesis.

From problem (10) we have $a^2 = \frac{P - f w}{2 A' T K^2}$

Then equations (4) and (5) become

$$\frac{P - f w}{2 A' T K^2} \cdot h^2 = b^2 (h w - \beta v)^2 \quad . \quad . \quad . \quad . \quad . \quad (1).$$

$$\beta b^2 (h w - \beta v)^2 = h^3 m A'' v^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

From (2) we have $\frac{v}{w} = \frac{h b \sqrt{\beta}}{b \sqrt{\beta^3} + \sqrt{m A'' h^3}}$ . . . . . (3)

From (1) we have

$$P h^2 - f h^2 w = 2 A' T K^2 b^2 \left\{ h w - \beta b \right\}^2$$

$$= 2 A' T K^2 b^2 \left\{ h^2 w^2 + \beta^2 v^2 - 2 h \beta w v \right\}$$

$$\therefore \left(\frac{P h^2}{w} - f h^2\right)\frac{1}{v} = 2 A' T K^2 b^2 \left\{ \frac{h^2 w}{v} + \frac{\beta^2 v}{w} - 2 h \beta \right\} \text{ from (3).}$$

$$= 2 A' T K^2 b^2 \left\{ \frac{h (b \sqrt{\beta^3} + \sqrt{m A'' h^3})}{b \sqrt{\beta}} + \frac{h b \beta^2 \sqrt{\beta}}{b \sqrt{\beta^3} + \sqrt{m A'' h^3}} - 2 h \beta \right\}$$

$$\therefore \left(\frac{P h}{w} - f h\right)\frac{1}{v} = 2 A' T K^2 b^2 \left\{ \frac{(b \sqrt{\beta^3} + \sqrt{m A'' h^3})^2 + b^2 \beta^3}{b^2 \beta^2 + b \sqrt{m A'' \beta h^3}} - 2 \beta \right\}$$

$$= 2 A' T K^2 b^2 \left\{ \frac{b^2 \beta^3 + m A'' h^3 + 2 b \sqrt{m A'' \beta^3 h^3} + b^2 \beta^3 - 2 b^2 \beta^3 - 2 b \sqrt{m A'' \beta^3 h^3}}{b (b \beta^2 + \sqrt{m A'' \beta h^3})} \right\}$$

$$= \frac{2 A' T K^2 b m A'' h^3}{b \beta^2 + \sqrt{m A'' \beta h^3}}$$

$\therefore P - f w = \frac{2 A' T K^2 b m A'' h^2}{b \beta^2 + \sqrt{m A'' \beta h^3}} \times v w$. Substitute the value of w in this equation.

$$\therefore P = f \frac{(b \beta^2 + \sqrt{m A'' \beta h^3})}{h b \beta} v + \frac{2 A' T K^2 m A'' h}{\beta} \cdot v^2 \quad . \quad . \quad . \quad (4).$$

Substitute the value of v in the above equation, we have

$$P = f w + \frac{2 A' T K^2 b^2 m A'' h^3 \beta}{(b \beta^2 + \sqrt{m A'' \beta h^3})^2} \cdot w^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5).$$

Equation (4) shows that the force applied to move the engine varies partly as the velocity, and partly as the square of the velocity, with the same screw.

The same remark applies to equation (5).

Since $\frac{v}{w} = \frac{h\,b\,\beta}{b\,\beta^2 + \sqrt{m\,A''\,\beta\,h^3}}$ we have

$$S = p - \frac{2\,\pi\,h\,b\,\beta}{b\,\beta^2 + \sqrt{m\,A''\,\beta\,h^3}} = p - \frac{2\,\pi\,b}{b\left(\frac{\beta}{h}\right) + \sqrt{m A''\left(\frac{h}{\beta}\right)}}$$

$$= p - \frac{2\,\pi\,b\,r}{b\,\cot.\,A + r\sqrt{m\,A''\,r\,\tan.\,A}}$$

by substituting the value of b from problem (10).

$$S = p - \frac{2\,\pi\,\sqrt{n}\,.\,\sqrt{h}\,.\,r^2\,\tan.\,A\,\sqrt{1 + 2\tan.^2 A.\log.(\sin.\,A)}}{2\,\pi\,.\,\sqrt{n}\,.\,\sqrt{h}\,.\,r\,\sqrt{1 + 2\tan.^2 A.\log.(\sin.\,A)} + \sqrt{m\,A''\,r\,\tan A}} \quad . \quad (6).$$

$$\therefore s = p - \frac{r\,\tan.\,A}{1 + \frac{\sqrt{m\,A''\tan.\,A}}{2\,\pi\,.\,\sqrt{n}\,\sqrt{1 + 2\tan.^2 A.\log.(\sin.\,A)}\,.\,\sqrt{h\,r}}} \quad . \quad . \quad , \quad (7)$$

Now if the angle of a screw be constant, the slip varies between p and $p - r$ tan. A: this is evident from formula (7). Hence a vessel cannot pass over a space greater than r tan. A during one revolution of the screw, when the screw is working in still water.

Formula (6) shows that the slip is independent of the power applied, and the friction of the engine; neither of which has any control over the slip.

Equation (4), problem (13), will take the following form:

$$P = f\left(\frac{\cot.\,A}{r} + \frac{\sqrt{m\,A''\,r\,\tan.\,A}}{b}\right)v + 2\,A'\,T\,K^2\,m\,A''\,r\,\tan.\,A.\,v^2$$

$$= f\left(\frac{\cot.\,A}{r} + \frac{\sqrt{m\,A''\,r\,\tan.\,A}}{\sqrt{n}\,.\,\sqrt{h}\,.\,r^2\tan.\,A\,\sqrt{1 + 2\tan.^2 A.\log.(\sin.\,A)}}\right)v$$

$$+ \frac{\frac{T\,m\,A''\,h\,r^4}{4}\left\{(2 + \tan.^2 A)\,\mathrm{cosec.}\,A - \tan.^3 A\log.\left(\cot.\,\frac{A}{2}\right)\right\}v^2}{\cot.\,A} \quad . \quad . \quad (8.)$$

PROBLEM XIV.

To find the Dimensions of the Screw when the Slip is a Minimum.

Since $2\,\pi\,r$ tan. $A = p$, equation (6) of last problem becomes

$$S = 2\,\pi\,r\,\tan.\,A - \frac{2\,\pi\,.\,\sqrt{n}\,.\,r^2\,\sqrt{h}\,.\,\tan.\,A\,\sqrt{1 + 2\,\tan.^2 A\log.(\sin.\,A)}}{2\,\pi\,.\,\sqrt{n}\,.\,r\,\sqrt{h}\,\sqrt{1 + 2\tan.^2 A\log.(\sin.\,A)} + \sqrt{m\,A''\,r\,\tan.\,A}}$$

This may be made a minimum by finding the values of h, r, and the angle A from the three equations.

$$\frac{d\,S}{d\,h} = 0\,;\ \frac{d\,S}{d\,r} = 0\,;\ \frac{d\,S}{d\,A} = 0$$

There is no difficulty in this problem, further than the complex nature of the functions to be differentiated, and the troublesome nature of the equations to be determined.

PROBLEM XV.

To find the Dimensions of the Screw, when the Power applied is a Minimum, to produce a given Velocity.

From equation (8), problem (13), we have

$$\frac{P}{v f} = \frac{\cot A}{r} + \frac{\sqrt{m A'' r}}{\sqrt{n} \cdot \sqrt{h} \cdot r^2 \sqrt{\tan. A + 2 \tan.^3 A \log. (\sin. A)}}$$

$$+ \frac{v T m A'' h r^4}{4 f} \left\{ (2 \tan. A + \tan.^3 A) \operatorname{cosec.} A - \tan.^4 A \log. \left(\cot. \frac{A}{2} \right) \right\} \quad . \quad (1),$$

Hence P is a minimum, when

$$V = \frac{\cot. A}{r} + \frac{\sqrt{m A'' r}}{\sqrt{n} \cdot \sqrt{h} \cdot r^2 \sqrt{\tan. A + 2 \tan.^3 A \log. (\sin. A)}}$$

$$+ \frac{v T m A'' h r^4}{4 f} \left\{ (2 \tan. A + \tan.^3 A) \operatorname{cosec.} A - \tan.^4 A \log. \left(\cot. \frac{A}{2} \right) \right\}$$

is a minimum. By differentiating this equation we have

$$\frac{d V}{d r} = - \frac{\cot. A}{r^2} - \frac{3 \sqrt{m A'' r}}{2 \sqrt{n} \cdot \sqrt{h} \cdot r^3 \sqrt{\tan. A + 2 \tan.^3 A \log. (\sin. A)}}$$

$$+ \frac{v T m A'' h r^3}{f} \left\{ (2 \tan. A + \tan.^3 A) \operatorname{cosec.} A - \tan.^3 A \log. \left(\cot. \frac{A}{2} \right) \right\} = 0 \quad . \quad (2).$$

$$\frac{d V}{d h} = - \frac{\sqrt{m A'' r}}{2 \sqrt{n} \cdot \sqrt{h^3} \cdot r^2 \sqrt{\tan. A + 2 \tan.^3 A \log. (\sin. A)}}$$

$$+ \frac{v T A'' r^4}{4 f} \left\{ (2 \tan. A + \tan.^3 A) \operatorname{cosec.} A - \tan.^4 A \log. \left(\cot. \frac{A}{2} \right) \right\} = 0 \quad . \quad . \quad (3).$$

$$\frac{d V}{d A} = - \frac{\operatorname{cosec.}^2 A}{r} - \frac{\sqrt{m A'' r} \left(\sec.^2 A + 6 \tan.^2 A \sec.^2 A \log. (\sin. A) + 2 \tan.^2 A \right.}{2 r^2 \sqrt{n h} \left\{ \tan. A + 2 \tan.^3 A \log. (\sin. A) \right\}^{\frac{3}{2}}}$$

$$+ \frac{v T A'' h r^4 \sin. A}{f \cos.^5 A} \left\{ \cos. A - \sin.^2 A \log. \left(\cot. \frac{A}{2} \right) \right\} = 0 \quad . \quad . \quad . \quad . \quad (4).$$

From these three equations we may determine the values of h, r, and the angle A, so that V shall be a minimum.

If we take the angle and the length constant, then, from equation (2), we shall see that the power applied varies as $\frac{L}{r} + \frac{M}{r^{\frac{3}{2}}} + N r^4$, where L, M, N are constants.

If we take the angle and the diameter of the screw constant, then the power applied will vary as $\frac{L'}{\sqrt{h}} + M' h$, where L' and M' are constants.

In the first case, to obtain the value of the diameter, which gives the least possible force, we must determine r from (2).

$$\frac{v\,T\,m\,A''\,h\,r^3}{f}\left\{(\tan. A + \tan.^3 A)\operatorname{cosec.} A - \tan.^4 A \log.\left(\cot.\frac{A}{2}\right)\right\} = \frac{\cot. A}{r^2}$$

$$+ \frac{3\sqrt{m\,A''\,r}}{2\sqrt{n}\,.\sqrt{h}\,.\,r^3\sqrt{\tan. A + 2\tan.^3 \log.(\sin. A)}}$$

$$\therefore\ \frac{v\,T\,m\,A''\,h\,r^6}{f}\left\{(\tan. A + \tan.^3 A)\operatorname{cosec.} A - \tan.^4 A \log.\left(\cot.\frac{A}{2}\right)\right. = r \cot. A$$

$$+ \frac{3\sqrt{m\,A''} \times \sqrt{r}}{2\sqrt{n}\,.\sqrt{h}\sqrt{\tan. A + 2\tan.^3 \log.(\sin. A)}}$$

$$\therefore\ \frac{v\,T\,m\,A''\,h}{f}\left\{(\tan. A + \tan.^3 A)\operatorname{cosec.} A - \tan.^4 A \log.\left(\cot.\frac{A}{2}\right)\right\} r^4 = \cot. A$$

$$+ \frac{3}{2}\left\{\frac{m\,A''}{n\,h\,r(\tan. A + 2\tan.^3 \log.(\sin. A)}\right\}^{\frac{1}{2}}$$

An equation from which the value of r may be approximated to when the constants are given.

In the second case, the value of the length may be obtained from equation (3).

$$\frac{T\,m\,A''\,v\,r^4}{f}\left\{\left(\tan. A + \frac{\tan.^3 A}{2}\right)\operatorname{cosec.} A - \frac{\tan.^4 A}{2}\log.\left(\cot.\frac{A}{2}\right)\right\}$$

$$= \frac{\sqrt{m\,A''\,r}}{\sqrt{n\,h^3}\,r^2\sqrt{\tan. A + 2\tan.^3 \log.(\sin. A)}}$$

From which we obtain

$$\sqrt{h^3} = \frac{f\sqrt{r}}{T\sqrt{m\,A''\,n}\,.\,v\,r^6\sqrt{\tan. A + 2\tan.^3 \log.(\sin. A)}\left\{\left(\tan. A + \frac{\tan.^3 A}{2}\right)\operatorname{cosec.} A - \frac{\tan.^4 A}{2}\log.\left(\cot.\frac{A}{2}\right)\right\}} \quad . \quad . \quad (6).$$

From this equation h may be obtained when the constants are given.

NOTES ON CHAPTER II.

NOTE (1).

(1.) By referring to formula (2), problem (7), we have

$$F = 2\,n\,h\int_0^r f(V)\,x\,d\,x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1).$$

where F is the moment of accelerating force of both blades of the screw.

$\therefore\ \frac{P}{2\,A'\,T\,K^2} - F =$ accelerating force to produce angular motion in the screw . . . . (2).

Again—

Because $\cos.\ \theta = \frac{\beta x}{\sqrt{\beta^2 x^2 + h^2}}$ see cor. (1) prob. (6), we shall have

$\frac{\beta n \mathrm{f}(V)\, x}{\sqrt{\beta^2 x^2 + h^2}}$ = accelerating force in the direction of the vessel at a point x distance from the axis.

And since all the points in the helix, whose length is $\sqrt{\beta^2 x^2 + h^2}$, are situated at the same distance x from the axis of the screw, we shall have

$\beta n \mathrm{f}(V)\, x$ = accelerating force in the direction of the axis of the screw, of the helix x distance from the axis.

Then $F' = 2\beta n \int_0^r \mathrm{f}(V)\, x\, d x$ = accelerating force in the direction of the axis of the screw, of the helix x distance from the axis. . . (3).

Put R to represent the force in pounds avordupois, which is necessary to prevent the action of the screw-blades on the water from moving the vessel to which the screw is attached, when the engine is moving the screw with a uniform angular velocity w:

Then R will be the measure of the accelerating force of both blades of the screw in the direction of the axis of the screw.

Therefore we shall have, from equation (3),

$$R = 2\beta n \int_0^r \mathrm{f}(V)\, x\, d x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4).$$

Now, when the angular velocity of the screw is uniform, the accelerating force to drive the screw round must vanish; then equation (2) will become

$$\frac{P}{2 A' T K^2} = 2 n h \int_0^r \mathrm{f}(V)\, x\, d x \quad . \quad . \quad . \quad . \quad . \quad (5).$$

From the equations (4) and (5) we can eliminate the unknown integral $\int_0^r \mathrm{f}(V)\, x\, d x$, and obtain the important relation

$$R = \frac{\beta P}{2 A' T K^2 h}$$

$$\therefore P = \frac{2 A' T K^2 h}{\beta}$$

$$= \frac{A' T K^2 p R}{\pi} \quad \text{Since } \beta = \frac{2 h \pi}{p}$$

$A' T K^2 = I$, the moment of inertia of the screw blade . . (6).

$$\therefore P = \frac{I p R}{\pi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7).$$

From this singular and important equation we may readily obtain the

useful moment of the engine used to turn the screw-propeller round, by observing the value of R from experiment, and calculating the moment of inertia of the screw blade from problem (5).

If the same screw blade be used with two different moments of force, we shall have

$$P' = \frac{I p R'}{\pi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8).$$

From the equations (7) and (8) we shall have

$$\frac{P}{P'} = \frac{R}{R'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9).$$

This equation shows *this law,—that the moments of force exerted by the engine to drive the screw round are proportional to the forces necessary to prevent motion in the vessel in which the screw is fixed.*

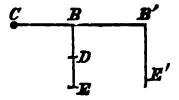
(2). Formula (7) supplies a practical method of obtaining the *useful moment* of the engine transmitted to the axis of the screw. This moment of the engine is an entirely different quantity from the *force* of the engine measured by the indicator.

There can be no *rotatory motion* without a *moment of force* to produce it. And since this *moment of force* is composed of two independent elements—namely, the force applied to turn the screw, and the distance from the point at which the force is applied to the axis of the screw,—*an engine may exert a large amount of force, and still have comparatively a small moment of force, which is alone effective in driving the screw blade round.*

From the testimony of the highest practical experience, and my own observations, I am led to believe that the distinction between *force* and *moment of force*, in estimating the capabilities of an engine to turn the screw blade, has not met with that mature consideration which its practical importance appears to demand.

(3). *A unit of moment of force may be represented by a pound, acting one foot from the axis of motion; therefore, P pounds acting one foot from the axis of motion will produce P units of moment of force.*

Thus, if C be a fixed point in the axis of motion, and $C B$ = one foot, and $B D$, at right angles to $B C$, represents one pound:



Then $B C \times B D$ = a unit of moment of force.

Let $B E$ represent Q pounds acting at B.

Then $Q \times B C = Q$ units of moment of force.

To find the units of moment of force which a force, represented by $B' E'$ acting at the distance $B' C$ from the axis of motion, will produce.

Let Υ represent the number of units of moment of force.

Then, by the principle of the lever we shall have

$$C B \times \Upsilon = B' C \times B' E' \quad . \quad . \quad . \quad (10).$$

Let the number of pounds acting at B' be represented by B, and the number of feet in $B' C$ be represented by b, so that, any capital denoting the force, the corresponding small letter will denote the distance from the axis to the point at which it is applied.

Thus $h H$ denotes the force of H pounds acting at h feet from the axis of motion, and is commonly called a *moment*.

Adopting this notation, and calling $C B =$ one foot, equation (10) will become

$$\Upsilon = b B \quad . \quad . \quad . \quad . \quad . \quad . \quad (11).$$

If there is any number of forces, B, C, D, &c. &c., acting to turn the screw round, then we shall have

$$\Upsilon = b B + c C + d D + \text{\&c. \&c.} \quad . \quad . \quad (12).$$

(4). Formula (7) affords a practical method of determining the *quality of an engine*, in terms of the useful moment exerted in turning the screw blades round. To ascertain the useful moment of force from this formula, we must determine the values of I and p, which depend only on the screw blade used to propel the vessel, and also the value of R, which may be called the *statical surface pressure* of the screw blades.

The value of p is given, and that of I can be readily obtained, in any screw used in practice, by the table in note (3): the value of R is obtained in the following manner.

Let a given pressure, measured by the indicator, be applied to the piston of the engine which is adopted to turn the screw round; if the vessel, to which the screw is attached, be free to move, the pressure of the screw blade on the water will move it; but if the vessel be not free to move, that is, let the vessel have a point in its stern connected with a fixed point, apart from the vessel, by means of a strong rope or chain, the tension of which is sufficient to prevent the vessel from moving in a horizontal direction, by the pressure of the screw blade on the water; then the engine will drive the screw round without putting the vessel in motion.

If the engine then be allowed to work till the screw has attained a uniform velocity, the tension of the rope or chain, fixed to the stern of the vessel as above described, will be the value of R in formula (7).

I shall not attempt to describe the various practical methods which may be successfully adopted to measure the tension of the rope or chain: this object will be best accomplished by practical men. The Dynamometer would measure the tension correctly, if the friction of the apparatus, and the bearings of the axle of the screw, could be properly ascertained.

(5). The quality of an engine to turn the screw round may be measured as follows:

If there are two engines, B and C, whose forces, as measured by the indicator, are represented by B and C, which drive the screw round in a

manner to produce statical *surface pressures* on the blade of the screw, represented by R_1 and R_2 respectively:

Now, if the same screw be used for both engines, the value of P, in formula (7), will be the measure of the number of units of useful *moment of force.* Hence we shall have

$$\Upsilon_1 = \frac{I\,p}{\pi} \times R_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13).$$

$$\Upsilon_2 = \frac{I\,p}{\pi} \times R_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

Therefore, if R_1, which is always in a constant ratio to Υ_1, is greater, equal to, or less than R_2: then the engine B has a greater, equal, or a less advantage than the engine C in driving the screw round.

(6). It will be convenient to adopt the following notation:

Υ_{ϖ} = the number of units of moments of force, when ϖ pounds pressure, as measured by the indicator, are applied to the piston.

R_{ϖ} = the surface pressure of the screw blade, where ϖ pounds pressure, as measured by the indicator, are applied to the piston.

Then we have $\Upsilon_{\varpi} = \frac{I\,p}{\pi} \,.\, \mathrm{R}_{\varpi}$ . . . . . (15).

The values of Υ_{ϖ}, when ϖ is taken for every pound pressure between the lowest and highest pressure used by the engine, should be obtained by making requisite experiments, and the value of I determined by taking a screw blade of uniform thickness.

These results tabulated, would be valuable, to show the number of useful moments of force when a certain pressure is applied to the piston, not only to ascertain the quality of the engine and screw; but also when the vessel is in motion we should be able to ascertain the units of moments of force applied to the screw to turn it round.

(7). The quality of screws may be obtained by using two different screws as follows:

$$\Upsilon_{\varpi} = \frac{I\,p}{\pi} .\ R_{\varpi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16).$$

$$\Upsilon'_{\varpi} = \frac{I'\,p'}{\pi} .\ R'_{\varpi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17).$$

Now, if Υ_{ϖ} be greater, equal to, or less than Υ'_{ϖ}, then the first screw has a greater, equal, or less advantage than the second; because the same number of useful moments of force is applied to turn the screw round in both cases, and the difference between Υ_{ϖ} and Υ'_{ϖ}, determined by experiment, can only arise from the fact, that one screw is better adapted than the other for the purpose of propulsion.

(8). The above relations do not depend upon the law that the resistances are proportional to the square of the normal velocity; the peculiar circumstance of the two equations, (4) and (5), involving the same integral depending upon the law connecting the force with the velocity, has enabled me to eliminate the integral, which is the unknown part, and establish a relation between the other elements of the equations; the only condition which is required, is the uniform motion of the screw.

If this condition be complied with, it is not of the slightest consequence, so far as the above relations are concerned, how the water in which the screw works is disturbed.

NOTE (2).

(1). Put R'_{ϖ} to represent the accelerating force to resist the motion of the vessel moving with a velocity v, and ϖ pounds pressure applied to the engine.

Then, by referring to problem (8), we shall have, from equations (2) and (3),

$\frac{\Upsilon \varpi}{2I} - 2nh\int_0^r \mathrm{f}(V - V')\, x\, dx =$ accelerating force to produce angular motion in the screw . . . . (1).

$2\beta n\int_0^r \mathrm{f}(V - V')\, x\, dx - R'_{\varpi} =$ accelerating force of both blades of the screw to produce motion in the vessel . . . . . . (2).

When the vessel and the screw have attained a uniform velocity, the accelerating forces are equal to zero.

Therefore the equations (1) and (2) will become

$$\frac{\Upsilon \varpi}{2I} = 2nh\int_0^r \mathrm{f}(V - V')\, x\, dx \quad . \quad . \quad . \quad . \quad . \quad . \quad (3).$$

$$R'_{\varpi} = 2\beta n\int_0^r \mathrm{f}(V - V')\, x\, dx \quad . \quad . \quad . \quad . \quad . \quad . \quad (4).$$

From these equations we can eliminate the portion depending upon the unknown law of resistances, which is the integral, and obtain a relation which is true, whatever may be the law connecting the force with the normal velocity.

$$\therefore R'_{\varpi} = \frac{\beta \Upsilon \varpi}{2hI}$$

Put $\beta = \frac{2h\pi}{p}$, then we shall have

$$R'_{\varpi} = \frac{\pi \Upsilon \varpi}{pI} \quad . \quad . \quad . \quad . \quad . \quad (5).$$

(2). If the same screw be used with a different power ϖ', we shall have

$$R'_{\varpi'} = \frac{\pi \Upsilon_{\varpi'}}{p I} \quad \ldots \ldots \ldots \ldots \quad (6).$$

$$\therefore \frac{R'_{\varpi}}{R'_{\varpi'}} = \frac{\Upsilon_{\varpi}}{\Upsilon_{\varpi'}} \quad \ldots \ldots \ldots \ldots \quad (7).$$

This equation shows the law,—that the resistances are proportional to the moments of force exerted to put the vessel in motion.

(3.) Equation (5) will supply a practical mode of determining the amount of resistance of a vessel to which the screw is attached, and thereby determine the *quality* of a vessel with respect to her capability of sailing.

In the equation (5) we have Υ_{ϖ} given by the experiments for every pound pressure on the piston, between the lowest and highest limits; hence, the value of R'_{ϖ} may be computed.

If we take two vessels, B and C, having two different engines and screws, and take ϖ pounds pressure on the piston in the engine B, and ϖ' pounds pressure on the piston in the engine C; so that the same uniform velocity is obtained in the two vessels:

$$\therefore R'_{\varpi} = \frac{\pi \Upsilon_{\varpi}}{p I} \quad \ldots \ldots \ldots \ldots \quad (8).$$

$$\text{and } R'_{\varpi'} = \frac{\pi \Upsilon_{\varpi'}}{p' I'} \quad \ldots \ldots \ldots \ldots \quad (9).$$

Then, if $R'\varpi$ be greater, equal to, or less than $R'\varpi'$; then the vessel B has less, equal, or greater sailing qualities than the vessel C. The same remark which is made in art. (8), note (1), will apply to all the results obtained in this note.

NOTE (3).

(1). From cor. (3), prob. (1), chap. (1), we have

$$\tan. A = \frac{p}{2 r \pi}$$

$$\therefore 2 \pi \tan. A = \left(\frac{p}{r}\right) \quad \ldots \ldots \ldots \quad (1).$$

This equation is solved for every half angle from 10° to 25° (which are supposed to include all angles used in practice) in the three first columns of the following Table (page 41). For instance, let the angle A be $13\frac{1}{2}°$, opposite to this angle we have 1·50846, which is the value of 2π tan. $(13\frac{1}{2}°)$, or its equal $\left(\frac{p}{r}\right)$. Column (3) is the differences of column (2), marked $\left(\frac{p}{r}\right)$.

(2). By note at page (9), we have the area of the screw blade.

$$A' = h\,r\left\{\frac{\text{cosec.}\,A + \tan.\,A.\,\log.\left(\cot.\frac{A}{2}\right)}{2}\right\}$$

If we put $\delta = \dfrac{\text{cosec.}\,A + \tan.\,A.\,\log.\left(\cot.\frac{A}{2}\right)}{2}$ we shall have

$$A' = h\,r\,\delta. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

In the following table (page 41) we have the values of

$\delta = \dfrac{\text{cosec.}\,A + \tan.\,A\,\log.\left(\cot.\frac{A}{2}\right)}{2}$ for every half angle from 10° to 25°.

(3). From cor. (2), prob. (5), page 19, we have

$$2\,A'\,K^2 = h\,r^3\left\{\frac{(2 + \tan.^2 A)\,\text{cosec.}\,A - \tan.^3 A\,\log.\left(\cot.\frac{A}{2}\right)}{4}\right\}$$

which is the moment of inertia of both blades of the screw.

Put $\delta' = \dfrac{(2 + \tan.^2 A)\,\text{cosec.}\,A - \tan.^3 A\,\log.\left(\cot.\frac{A}{2}\right)}{4}$ then we shall have

$$2\,A'\,K^2 = h\,r^3\,\delta'. \quad . \quad . \quad . \quad . \quad . \quad (3).$$

Column (6) of the following table gives the value of δ' for every half angle from 10° to 25°.

(4). From equations (2) and (3) we shall have

$$2\,K^2 = r^2\left(\frac{\delta'}{\delta}\right)$$

$$\therefore K = r\sqrt{\frac{\delta'}{2\,\delta}}$$

Put $\delta'' = \sqrt{\dfrac{\delta'}{2\,\delta}}$ then we shall have

$$K = r\,\delta'' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4).$$

The last column of the table gives the value of δ'' for every half angle from 10° to 25°.

(5). If the blade of the screw has a thickness T, then equation (3) will become

$2\,A'\,T\,s\,K^2 = T\,s\,h\,r^3\,\delta'$, the moment of inertia where s equals the weight of a unit of the blade.

Let W equal weight of both blades of the screw, and g equal the accelerating force of gravity.

Now $2\,A'\,T\,s$ being the mass in rotation, we shall have

$2\,A'\,T\,s = \frac{W}{g}$. See Professor De Morgan, Diff. and Integral Calculus, pages 476 and 477.

$$\therefore\ T\,s = \frac{W}{2\,g\,A'}$$

$$\therefore\ 2\,A'\,T\,s\,K^2 = \frac{W\,h\,r^3\,\delta'}{2\,g\,A'}$$

$$= W\,r^2 \times \left(\frac{\delta'}{2\,g\,\delta}\right)$$

Put $\delta''' = \frac{\delta'}{2\,g\,\delta}$, then we shall have

$$2\,A'\,T\,s\,K^2 = W\,r^2\,\delta''' \quad . \quad . \quad . \quad . \quad (5).$$

for the moment of inertia of both blades of the screw.

Example: the screw of the *Termagant* is

$$p = 18 \text{ feet},\ r = 7{\cdot}75 \text{ feet, and } h = 3 \text{ feet}.$$

$$\therefore\ \frac{p}{r} = \frac{18}{7{\cdot}75} = 2{\cdot}32258$$

Now, look down the column marked $\left(\frac{p}{r}\right)$ until you observe the ratio nearest to 2·32258, which is 2·34919. Therefore the angle of the screw is 20½°.

Then $2\,A' = 2\,h\,r\,\delta = 15{\cdot}5 \times 3 \times 1{\cdot}74743 = 81{\cdot}2554$ square feet, the area of both blades of the screw.

Then $K = r\,\delta'' = 7{\cdot}75 \times {\cdot}65626 = 5{\cdot}086$ feet, the radius of gyration.

If we take the weight of the screw to be three tons, we shall have the moment of inertia

$W\,r^2\,\delta''' = 2240 \times 3 \times 7{\cdot}75 \quad \times {\cdot}013379 = 5400$ lbs. avoirdupois.

In consequence of the ratio 2·34919 given in the table being a little greater than the ratio 2·32258, the angle of the screw is not quite so much as 20½°, but this is sufficiently near the truth for most practical purposes. We can readily obtain the correct angle to a minute, by taking proportional parts.

Take the ratio next less than 2·32258, which is 2·28689, corresponding to 20°.

$$\therefore\ 6230 : 30' :: 3569 : 17'.$$

Hence the angle is 20° 17′ nearly.

Instead of taking 1·77772 to find the area, we must deduct from it a number proportional to 17′, which is obtained by

${\cdot}03199 \times 17 \div 30 = {\cdot}01812. \qquad \therefore\ 1{\cdot}77772 - {\cdot}01812 = 1{\cdot}75960.$

$\therefore 2\,A' = 2\,h\,r\,\delta = 15{\cdot}5 \times 3 \times 1{\cdot}7596 = 81{\cdot}8214$ square feet, the area.

To find the radius of gyration, we must take $\cdot 65766 - \cdot 65626 = \cdot 00140$ $\therefore \cdot 00140 \times 17 \div 30 = .00079$, which is the number to deduct from $\cdot 65766$.

$\therefore K = r\,\delta = 7{\cdot}75 \times \cdot 65687 = 5{\cdot}0907$ feet.

To find the moment of inertia we must take

$\cdot 013436 - \cdot 013379 = \cdot 000057$ $\therefore \cdot 000057 \times 17 \div 30 = 000032$, which is the number to be deducted from $\cdot 013436$.

Then $W r^2 \delta''' = 2240 \times 3 \times 7{\cdot}75^2 \times \cdot 013404 = 5410$ lbs. avoirdupois.

TABLE *for readily computing the Angle, Area, Moment of Inertia, and Radius of Gyration of the* SCREW PROPELLER; *the Radius, Pitch, and Length being given.*

Angle of Screw.	$\left(\frac{p}{r}\right)$	Diff.	The value of δ.	Diff.	The value of δ'.	Diff.	The value of δ''.	The value of δ'''.
10°	1·10789	·05663	3·09417		2·92080		·68701	·014662
10½	1·16452	·05681	2·96492	·12925	2·78680	·13400	·68553	·014599
11	1·22133	·05700	2·84788	·11704	2·66560	·12120	·68410	·014538
11½	1·27833	·05720	2·74199	·10589	2·55500	·11060	·68257	·014473
12	1·33553	·05742	2·64429	·09770	2·45378	·10122	·68115	·014413
12½	1·39295	·05764	2·55527	·08902	2·36085	·09293	·67967	·014350
13	1·45059	·05787	2·47344	·08183	2·27526	·08559	·67818	·014287
13½	1·50846	·05811	2·39799	·07545	3·19616	·07910	·67669	·014224
14	1·56657	·05837	2·32824	·06975	2·12289	·07327	·67520	·014162
14½	1·62494	·05863	2·26358	·06466	2·05483	·06806	·67371	·014100
15	1·68357	·05891	2·20349	·06009	1·99144	·06339	·67223	·014038
15½	1·74248	·05920	2·14753	·05596	1·93229	·05915	·67073	·013975
16	1·80168	·05948	2·09531	·05222	1·87698	·05531	·66925	·013913
16½	1·86116	·05980	2·04647	·04884	1·82515	·05183	·66777	·013852
17	1·92096	·06012	2·00071	·04576	1·77649	·04866	·66630	·013791
17½	1·98108	·06045	1·95777	·04294	1·73073	·04576	·66484	·013731
18	2·04153	·06079	1·91739	·04038	1·68764	·04309	·66339	·013671
18½	2·10232	·06115	1·87939	·03800	1·64698	·04066	·66194	·013611
19	2·16347	·06152	1·84355	·03584	1·60857	·03841	·66050	·013552
19½	2·22499	·06190	1·80971	·03384	1·57223	·03634	·65908	·013494
20	2·28689	·06230	1·77772	·03199	1·53781	·03442	·65766	·013436
20½	2·34919	·06270	1·74743	·03029	1·50517	·03264	·65626	·013379
21	2·41189	·06312	1·71872	·02871	1·47417	·03100	·65487	·013322
21½	2·47501	·06356	1·69149	·02723	1·44470	·02947	·65349	·013266
22	2·53857	·06401	1·66561	·02588	1·41466	·02804	·65166	·013192
22½	2·60258	·06447	1·64101	·02460	1·38995	·02671	·65077	·013156
23	2·66705	·06495	1·61760	·02341	1·36446	·02549	·64942	·013101
23½	2·73200	·06545	1·59529	·02231	1·34018	·02428	·64810	·013048
24	2·79745	·06596	1·57402	·02127	1·31696	·02322	·64679	·012996
24½	2·86341	·06649	1·55372	·02030	1·29477	·02219	·64550	·012944
25	2·92990		1·53433	·01939	1·27354	·02123	·64421	·012892

NOTE (4).

If we refer to note (2), chap. (2), page 37, we find

$\frac{\Upsilon_\varpi}{2I} - 2nh\int_0^r f(V - V')\,x\,dx$ = accelerating force to produce angular motion in the screw.

$2\beta n\int_0^r f(V - V')\,x\,dx - R'_\varpi$ = accelerating force of both blades of the screw to produce motion in the vessel.

By Whewell's Dynamics, pages 118, 185, part ii., we shall have

$$\frac{dw}{dt} = \frac{\Upsilon_\varpi}{2I} - 2nh\int_0^r f(V - V')\,x\,dx \quad . \quad . \quad . \quad . \quad . \quad (1).$$

$$\frac{dv}{dt} = 2n\beta\int_0^r f(V - V')\,x\,dx - R'_\varpi \quad . \quad . \quad . \quad . \quad . \quad (2).$$

Multiply (1) by β, and (2) by h, we shall have, by addition,

$$\frac{\beta\,dw}{dt} + \frac{h\,dv}{dt} = \frac{\beta\,\Upsilon_\varpi}{2I} - hR'_\varpi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3).$$

If the motion be uniform, $\frac{dw}{dt} = 0$ and $\frac{dv}{dt} = 0$.

$$\left.\begin{aligned} &\text{Hence } 2n\int_0^r f(V - V')\,x\,dx = \frac{\Upsilon_\varpi}{2hI} \\ &\text{And } 2n\int_0^r f(V - V')\,x\,dx = \frac{R'_\varpi}{\beta} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (4).$$

$$\therefore \frac{\Upsilon_\varpi}{2hI} = \frac{R'_\varpi}{\beta} = \frac{pR'_\varpi}{2h\pi}$$

$$\therefore R'_\varpi = \frac{\pi\,\Upsilon_\varpi}{pI} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5).$$

This result is the same as that which is obtained in note (2), page 37.

R'_ϖ being the measure of the accelerating force of resistance, we may take the relation

$$R'_\varpi = m\,A''\,v^2$$

where A'' is the area of the midship section, and m a constant quantity.

Substitute this value in (5), we shall have

$$v^2 = \frac{\pi\,\Upsilon_\varpi}{m\,A''\,p\,I} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6).$$

Hence this important theorem, ***The square of the velocity of the vessel is directly proportional to the moment of the engine, and inversely proportional to the rectangle of the pitch and moment of inertia.***

Cor. (1).—In the same vessel, with a different power and screw, we shall have

$$v_1^2 = \frac{\pi \Upsilon_{\varpi'}}{m A'' p' I'}$$

$$\therefore \frac{v^2}{v_1^2} = \frac{p' I' \Upsilon_{\varpi}}{p I \Upsilon_{\varpi'}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7).$$

If the same power be applied to the same vessel, with a different screw, we shall have

$$v_1^2 = \frac{\pi \Upsilon_{\varpi}}{m A'' p' I'}$$

$$\therefore \frac{v^2}{v_1^2} = \frac{p' I'}{p I} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8).$$

The following applications of the formula (7) show how nearly experiment agrees with the result which it enunciates. It will be seen, on referring to the table given by John Fincham, Esq. (see *History of Naval Architecture*, page 375), that the velocity is noted in feet per minute; the force of the engines is proportional to the moments, and the moments of inertia are assumed to be equal for the two screws used in the same ship, the diameters and lengths being nearly or quite equal.

The first illustration is obtained from the two trials of speed with the *Ajax*, the pitch of the screw having been altered.

From equation (7) we shall have, by assuming $I = I'$, which is nearly true in this case

$$\therefore \frac{v^2}{v_1^2} = \left(\frac{659}{720}\right)^2 = {\cdot}84$$

$$\text{and } \frac{p' \Upsilon_{\varpi}}{p \Upsilon_{\varpi'}} = \frac{17 \times 854}{19{\cdot}5 \times 820} = {\cdot}908$$

The results ·84 and ·908, which ought to be equal, have a difference only of ·068, or sixty-eight one-thousandths.

The next illustration is taken from the *Termagant's* trials:

$$\because \frac{v^2}{v_1^2} = \left(\frac{851}{964}\right)^2 = {\cdot}78$$

$$\text{and } \frac{p' \Upsilon_{\varpi}}{p \Upsilon_{\varpi'}} = \frac{\overset{\text{Ft.}}{17}\ ,,\ \overset{\text{in.}}{2\tfrac{1}{2}} \times 1124}{18 \times 1333} = {\cdot}8$$

The results ·78 and ·8 only differ in two-hundredths.

The last illustration is obtained from the experiments with the *Plumper*:

$$\therefore \frac{v^2}{v_1^2} = \left(\frac{753}{734}\right)^2 = 1{\cdot}05$$

$$\text{and } \frac{p' \Upsilon_{\varpi}}{p \Upsilon_{\varpi'}} = \frac{\overset{\text{Ft.}}{4},, \overset{\text{in.}}{6\tfrac{1}{2}} \times 148}{5 ,, 7 \times 139} = {\cdot}86$$

The small difference between theory and experiment, as shown in the above examples, may readily be accounted for by the assumption of equality of the moments of inertia.

CHAPTER III.

FURTHER DESCRIPTION OF THE SCREW PROPELLER.

The blade of the screw propeller is subjected to a strain by the pressure of the water against every point of its surface, during a revolution of its axis: the strain is greater on those parts of the screw near to the axis, and less on those at a greater distance from it. The remarks made by Professor Moseley, in his " Mechanical Principles of Engineering," page 532, appear to be so applicable to this case, that I cannot do better than quote them.

" The strongest form which can be given to a solid body, in the formation of which a given quantity of material is to be used, and to which the strain is to be applied under given circumstances, is that form which *renders it equally liable to rupture at every point.* So that when, by increasing the strain to its utmost limit, the solid is brought into that state bordering upon rupture at that point, it may be in the state bordering upon rupture at every other point. For let it be supposed to be constructed of any other form, so that its rupture may be about to take place at one point when it is not about to take place at another point, then may a portion of the material evidently be removed from the first point without placing the solid there *in* the state bordering upon rupture, and added to the second point, so as to take it *out* of the state bordering upon rupture at that point; and thus the solid, being no longer in the state bordering upon rupture at any point, may be made to bear a strain greater than that which was before upon the point of breaking it, and will have been rendered stronger than it was before.

" The first form was not therefore the strongest form of which it could have been constructed with the given quantity of material; nor is any form

the strongest which does not satisfy the condition of *an equal liability to rupture at every point.*

"The solid, constructed of the strongest form, with a given quantity of material, so as to be of given strength under a given strain, is evidently that which can be constructed, of the same strength, with the least material; so that the *strongest form* is also the form of the *greatest economy of material.*"

To accomplish the object which is admirably explained in this quotation, it will be necessary to increase the thickness of the blade from the *outer* extremity to the axis of the screw propeller; and for this purpose it will be convenient to measure the thickness of the screw blade in the direction of its axis, and not in a *normal* direction; the latter thickness will be readily obtained from the former.

Let $x = CB'$, fig. (3, *Plate*), as before, which is the distance of a helix $B'G'$ from the axis of the screw CE.

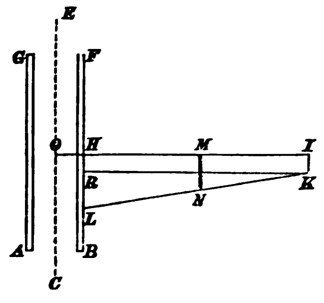
$T_x =$ thickness of the blade at x distance from the axis.

$A_x =$ the area of the section, at x distance from the axis, made by a concentric cylinder whose radius is x.

Generally, T_x is a function of x which may be conveniently written.

$$T_x = F(x) \quad . \quad . \quad . \quad . \quad (1).$$

Let $ABFG$ be the boss of the screw, and CE its axis; bisect BF in H, and let $HIKL$ represent the vertical section of the blade at half its length. Make OM equal to x, then MN perpendicular to OM is the thickness T_x.



If the upper surface be conoidal, OI is a straight line; and if the blade have a uniform thickness, KR, parallel to OI, will be a straight line on the under side of the blade; but if the thickness increases from I to H, then the under side KL is generally a curved line, which may be called the *directing curve of thickness.*

PROBLEM I.

To find the Sectional Area A_x.

$BGH'K$, fig. (2, *Plate*), will conveniently represent the development of the section at the extremity of the blade, and GI. sec. A its thickness parallel to the axis.

The area of $BGH'K = BG \cdot GI = BG \times \dfrac{T_r}{\text{Sec. } A} = BF \times T_r$ fig. (3, *Pl.*) (1).

Since $B F = r \beta$, we shall have the area of the section at the outer extremity of the blade equal to

$$\beta r T_r$$

Hence $A_x = \beta x F(x)$ . . . . (2).

Where $F(x)$ must be determined to satisfy the two conditions; first, the blade must not break in one place in preference to another; secondly, the blade must be strong enough to withstand the pressure to which it is subjected.

Cor. (1).—By multiplying equation (2) by $d x$, and integrating between the limits $x = r$, and $x = r_1$, we shall have

$\beta \int_{r_1}^{r} x \, F(x) \, d x$ = the solidity or volume of each blade of the screw. . . . . (3).

$\beta \int_{r_1}^{r} x^3 \, F(x) \, d x$ = moment of inertia of each blade of the screw. . . . . (4).

$\dfrac{\int_{r_1}^{r} x^2 \, F(x) \, d x}{\int_{r_1}^{r} x \, F(x) \, d x}$ = the distance of the centre of gravity of each blade from the axis of the screw. . . . . (5).

And if φ represents the strength of a unit of metal of which the screw is composed, then we shall have

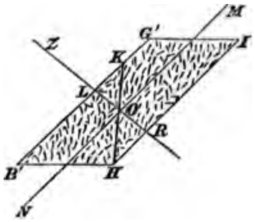
$\beta \, \varphi \, x \, F(x)$ = the strength of the blade at x distance from the axis. . . . . (6).

PROBLEM II.

To find the Neutral Line in the blade of the Screw subjected to a transverse strain.

In the solution of this question I shall avail myself of the researches of Professor Hodgkinson, F.R.S., &c. &c., as given in his Treatise on the Strength and other Properties of Cast Iron, pages 483 to 494 inclusive.

Fig. (1).



The development of the section of the screw blade, at x distance from the axis, is the parallelogram $B' G' H I$, where $H K$ is the thickness of the screw blade, parallel to the axis of the screw, and perpendicular to $B' H$; and $B' G'$ is the helix on the extended side, and $H I$ the helix on the compressed side of the blade.

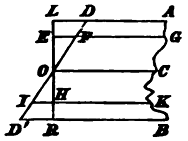
$N\,M$ is the neutral line, the position of which is required.

Draw $O\,z$ perpendicular to $N\,M$; and put $O\,L = a$, and $O\,R = a'$.

$$\therefore\ a + a' = L\,R = K\,H \text{ Cos. } K\,B'\,H = T_x \text{ Cos. } A_1 \quad . \quad . \quad (1).$$

Let fig. (2) be a section through $L\,R$ fig. (1), perpendicular to $N\,M$, the neutral line; by deflection, the fibre on the top of the blade $A\,L$ is extended the distance $L\,D$, while the fibre at the distance $O\,E$ from the neutral line is only extended $E\,F$, &c. &c., for any other fibre on the same side of $O\,C$; and the fibre on the bottom of the blade is compressed through the distance $R\,D'$, while the fibre at the distance $O\,H$ from the neutral line is only compressed $H\,I$, &c. &c., for any other fibre on the same side of $O\,C$.

Fig. (2).



Let S = sum of the forces required to elongate the whole of the fibres on the extended portion of the section, fig. (1).

and S' = sum of the forces required to compress the whole of the fibres on the compressed portion of the section, fig. (1).

The position of the neutral line is then determined by the equation

$$S = S' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

See Professor Hodgkinson on the Strength and other Properties of Cast Iron, page 486.

Let P_x = force in lbs. necessary to elongate a bar x feet, the bar being one foot long, and the section containing one square inch.

$\therefore\ A\,P_x$ = force in lbs. necessary to elongate a bar x feet, the bar being one foot long, and the section containing A square inches.

And $\dfrac{A\,P_x}{L}$ = force in lbs. necessary to elongate a bar x feet, the bar being L feet long, and the section containing A square inches.

Now if we suppose that P_x and x are connected by the relation

$$P_x = C\,\phi(x) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3),$$

where c is a constant quantity to be determined by experiment; substitute this value of P_x in the above formula, we shall have

$\dfrac{c\,A\,\phi\,(x)}{L}$ = force in lbs. necessary to elongate a bar x feet, the bar being L feet long, and the section containing A square inches. . . . . (4).

To apply this formula to the determination of the neutral line, we shall adopt the following notation.

Let $O\,z$, $O\,M$, fig. (1), be rectangular co-ordinate axes, $O\,z$ the axis of z, and $O\,M$ the axis of y.

So that $y\,z$ are the rectangular co-ordinates of the element of area $d\,y\,d\,z$. $A = d\,y\,d\,z$.

$L = (r - x)$; r being radius of blade.

Put $L\,D = \delta$, and $R\,D' = \delta'$, fig. (2).

$\therefore a : \delta :: z : x$, in equation (4), $= \frac{\delta\,z}{a}$

$\therefore \frac{c}{r-x}\,\phi\left(\frac{\delta\,z}{a}\right) d\,z\,d\,y$ = element of force, z distance from the neutral line.

$$\therefore S = \frac{c}{r-x}\iint \phi\left(\frac{\delta\,.\,z}{a}\right) d\,z\,d\,y \quad . \quad . \quad . \quad . \quad (5).$$

$$\text{and } S' = \frac{c'}{(r-x)}\iint \phi'\left(\frac{\delta'\,.\,z}{a'}\right) d\,z\,d\,y \quad . \quad . \quad . \quad . \quad (6).$$

$$\therefore S = \frac{c\,.\,B'\,G'}{r-x}\int \phi\left(\frac{\delta\,.\,z}{a}\right) d\,z \quad . \quad . \quad . \quad . \quad . \quad (7).$$

$$\text{and } S' = \frac{c'\,.\,B'\,G'}{r-x}\int \phi'\left(\frac{\delta'\,.\,z}{a'}\right) d\,z \quad . \quad . \quad . \quad . \quad . \quad (8).$$

c and c' may be obtained from experiment.

$$P_\delta = c\,\phi\,(\delta), \text{ and } P_{\delta'} = c'\,\phi'\,(\delta')$$

$$\therefore c = \frac{P_\delta}{\phi\,(\delta)} \text{ and } c' = \frac{P_{\delta'}}{\phi'\,(\delta')} \quad . \quad . \quad . \quad . \quad . \quad . \quad (9).$$

It will be necessary to omit the term used by Professor Hodgkinson to denote the defect of elasticity, in consequence of that defect not being established in the metal of which the screw blades are composed.

We shall assume therefore the relations

$$P_x = c\,x \text{ for extension,}$$

$$\text{and } P_x = c'\,x \text{ for compression.}$$

$$\therefore c = \frac{P_\delta}{\delta} \text{ and } c' = \frac{P_{\delta'}}{\delta'}$$

$$\text{and } S = \frac{B'\,G'\,.\,a\,.\,P_\delta}{2\,(r - x)}$$

$$\text{and } S' = \frac{B'\,G'\,a'\,.\,P_{\delta'}}{2\,(r - x)}$$

$$\therefore \frac{a}{a'} = \frac{P_{\delta'}}{P_\delta} \text{ from equation (2).}$$

$$\text{or } \frac{a}{a'} = \frac{c'\,\delta'}{c\,\delta} \quad . \quad . \quad . \quad . \quad . \quad (10).$$

where c and c' are the forces necessary to double a bar of metal one foot long and the section one inch square, they are commonly called the moduli of elasticity. But, from fig. (2), we must always have

$$O\,L : L\,D :: O\,R : R\,D'$$

$$\therefore a : \delta :: a' : \delta', \text{ or } \frac{a'}{a} = \frac{\delta'}{\delta}$$

Substitute this in (10), and we shall have

$$\frac{a^2}{a'^2} = \frac{c'}{c} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11).$$

The values of a and a', from equations (1) and (11), will be

$$a = \frac{\sqrt{\frac{c'}{c}} \times T_x \,.\, \cos.\, A_1}{1 + \sqrt{\frac{c'}{c}}} = \frac{\sqrt{\frac{c'}{c}} \times x\, T_x}{\left(1 + \sqrt{\frac{c}{c}}\right)\sqrt{x^2 + r^2 \tan.^2 A}} \quad . \quad (12).$$

$$a' = \frac{T_x \,.\, \cos.\, A_1}{1 + \sqrt{\frac{c'}{c}}} = \frac{x\, T_x}{\left(1 + \sqrt{\frac{c'}{c}}\right)\sqrt{x^2 + r^2 \tan.^2 A}} \quad . \quad . \quad . \quad . \quad (13).$$

PROBLEM III.

To find the Moments of Forces of Extension and Compression.

From the last problem we shall have

$$M = \frac{c}{r-x}\iint \phi\left(\frac{\delta\, z}{a}\right) z\, dz\, dy = \text{moment of forces of extension.}$$

$$M' = \frac{c'}{r-x}\iint \phi'\left(\frac{\delta'\, z}{a'}\right) z\, dz\, dy = \text{moment of forces of compression.}$$

Integrating between proper limits, we have

$$M = \frac{c\,.\,B'\,G'}{r-x}\int_0^a \phi\left(\frac{\delta\, z}{a}\right) z\, dz \quad . \quad . \quad . \quad . \quad . \quad (1).$$

$$M' = \frac{c'\,.\,B'\,G'}{r-x}\int_0^{a'} \phi'\left(\frac{\delta'\, z}{a'}\right) z\, dz \quad . \quad (2).$$

If we take $\phi\left(\frac{\delta\, z}{a}\right) = \frac{\delta\, z}{a}$, and $\phi'\left(\frac{\delta'\, z}{a'}\right) = \frac{\delta'\, z}{a'}$, that is, assume, as in the last problem, the elasticity to be perfect,

$$\therefore M + M' = \frac{c \,.\, B' G' \,.\, \delta}{a\,(r - x)} \int_0^a z^2\, d z + \frac{c' \,.\, B' G' \,.\, \delta'}{a'\,(r - x)} \int_0^{a'} z^2\, d z$$

$$= \frac{c \,.\, B' G'\, \delta\, a_2}{3\,(r - x)} + \frac{c' \,.\, B' G'\, \delta'\, a'^2}{3\,(r - x)}$$

$$= \frac{B' G' \,.\, c\, a^2}{3\,(r - x)}\,(\delta + \delta')$$

$$= \frac{B' G' \,.\, c\, a^2}{3\,(r - x)}\left(\delta + \frac{\delta\, a'}{a}\right)$$

$$= \frac{B' G' \,.\, c\, a\, \delta}{3\,(r - x)}\,(a + a')$$

$$= \frac{B' G' \,.\, c\, a\, \delta}{3\,(r - x)}\; T_x \;\cos.\, A_1$$

$$\therefore M + M' = \frac{\beta\, \delta\, \sqrt{c\, c'} \,.\, x\, T_x^2 \;\cos.\, A_1}{3\,(r - x)\left(1 + \sqrt{\frac{c'}{c}}\right)} - \frac{\beta^2\, \delta\, \sqrt{c\, c'} \,.\, x^2\, T_x^2}{3\,(r - x)\left(1 + \sqrt{\frac{c'}{c}}\right)\sqrt{\beta^2 x^2 + h^2}} \quad (3).$$

PROBLEM IV.

To find the Moment of Force exerted by the Pressure of the Water on the Blade of the Screw, to break it at the distance x from its Axis.

The moments required are the sum of the moments between the section at $B'\ G'$ and the extremity of the blade $B\ G$. Fig. (3, *Plate.*)

Take a variable helix at X distance from B', whose length is

$$\sqrt{\beta^2\,(x + X)^2 + h^2}$$

By problem (6), chapter (2), the velocity perpendicular to the helix is

$$V = \frac{h\, w\,(x + X)}{\sqrt{\beta^2\,(x + X)^2 + h^2}}$$

and $n\, \mathrm{f}\,(V)$ = force at a point in the helix.

Consequently, $n\, \mathrm{f}\,(V)\, X$ = moment of force at a point in the helix, in consequence of every point in the helix being the same distance X from the section at B'.

$$\therefore N = n \int_0^X X\, \mathrm{f}\,(V)\, \sqrt{\beta^2\,(x + X)^2 + h^2}\; d X \quad . \quad . \quad (1).$$

Now the motive force, or the pressure on the blade of the screw, may be taken to be proportional to the velocity;

$$\therefore N = h\, n\, w \int_0^X X\,(x + X)\; d X$$

$$= h\, n\, w \int_0^X (x\, X + X^2)\; d X$$

$$= h\,n\,w\left(\frac{x\,X^2}{2} + \frac{X^3}{3}\right);\ \text{from } X = 0 \text{ to } X = (r - x)$$

$$= \frac{h\,n\,w}{6}\,(r - x)^2\,(3\,x + 2\,(r - x))$$

$$\therefore\ N = \frac{h\,n\,w\,(r - x)^2\,(x + 2\,r)}{6}\ .\ .\ .\ .\ .\ .\ .\ (2).$$

Hence, by formula (2), page 47, we shall have

$$\frac{\beta^2\,\delta\,\sqrt{c\,c'}\,x^2\,T_x^2}{3\,(r - x)\left(1 + \sqrt{\frac{c'}{c}}\right)\sqrt{\beta^2\,x^2 + h^2}} = \frac{h\,n\,w\,(r - x)^2\,(x + 2\,r)}{6}$$

$$\therefore\ T_x^2 = \frac{h\,n\,w\left(1 + \sqrt{\frac{c'}{c}}\right)}{2\,\beta^2\,\delta}\ \frac{(r - x)^3\,(x + 2\,r)\sqrt{\beta^2\,x^2 + h^2}}{x^2}\ .\ .\ (3).$$

which is the equation to the *directing curve of thickness.*

Cor. (1).—If we measure the thickness perpendicular to the blade, and call this thickness T'_x,

$$\therefore\ T_x\ \cos.\ A_1 = T'_x$$

$$\text{or } T_x = T'_x \times \frac{\sqrt{\beta^2\,x^2 + h^2}}{\beta\,x}$$

$$\therefore\ T'^2_x = \frac{h\,n\,w\left(1 + \sqrt{\frac{c'}{c}}\right)}{2\,\delta}\ \frac{(r - x)^3\,(x + 2\,r)}{\sqrt{\beta^2\,x^2 + h^2}}\ .\ .\ .\ .\ .\ (4).$$

PROBLEM V.

To find the Volume of the Screw Blade when the Curve of Thickness is the Parabolic Order.

By referring to prob. (1) we have, from cor. (1),

$$S = \beta\int_{r_1}^{r} x\,F(x)\,d\,x$$

$$\text{Let } F(x) = B_1 + B_2\,x + B_3\,x^2 + B_4\,x^3 + \&c.\ \&c.\ .\ .\ (1).$$

$$\therefore\ S = \beta\int_{r_1}^{r}(B_1\,x + B_2\,x^2 + B_3\,x^3 + B_4\,x^4 + \&c.)\,d\,x$$

$$= \beta\left(B_1\,\frac{(r^2 - r_1^2)}{2} + B_2\,\frac{(r^3 - r_1^3)}{3} + B_3\,\frac{(r^4 - r_1^4)}{4} + B_4\,\frac{(r^5 - r_1^5)}{5} + \&c.\ \&c.\right)$$

The constants B_1, &c. &c., may be determined to approximate to any given curve.

PROBLEM VI.

To find the Moment of Inertia of the Screw Blade when the Curve of Thickness is the Parabolic Order.

From cor. (1), prob. (1), we have

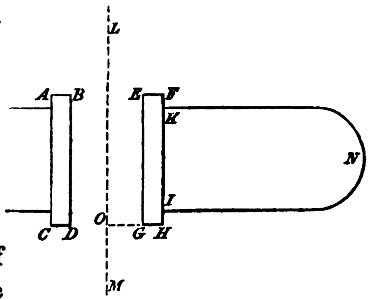
$$M K_1^2 = \beta \int_{r_1}^{r} x^3 \, F(x) \, dx$$

$$\text{Let } F(x) = B_1 + B_2 x + B_3 x^2 + B_4 x^3 + \text{\&c.}$$

$$\therefore M K_1^2 = \beta \int_{r_1}^{r} (B_1 x^3 + B_2 x^4 + B_3 x^5 + B^4 x^6 + \text{\&c.}) \, dx$$

$$\therefore 2 M K_1^2 = 2 \beta \left(B_1 \frac{(r^4 - r_1^4)}{4} + B_2 \frac{(r^5 - r_1^5)}{5} + B_3 \frac{(r^6 - r_1^6)}{6} + B_4 \frac{(r^7 - r_1^7)}{7} + \text{\&c. \&c.} \right)$$

Cor. (1).—The accompanying figure is a section of the boss of the screw blade, the plan of which is a circle, the boss being a concentric cylinder.



IKN is the screw blade, which is attached to the boss at the points I and K.

$KI = h$, the length of the blade.

$FH = h'$, the length of the boss.

$OH = r_1$.

$OG = r_2$.

LM is the axis of the cylinder, out of which the screw blade is formed. The portion $BDGE$ is a hollow cylinder, prepared for receiving the axis which extends through the vessel to the engine, and which is generally made of different metal from that which forms the boss and blades of the screw.

Let M_1 = mass of one blade of the screw.

M_2 = mass of the boss.

M_3 = mass of the axis in the boss of the screw.

And S_1 = weight of a cubic foot of the blade and boss.

S_2 = weight of a cubic foot of the axis.

$$\left\{ 2 M_1 + M_2 + M_3 \right\} K^2 = 2 M_1 K_1^2 + M_2 K_2^2 + M_3 K_3^2 \quad . \quad (1)$$

See page 18, equation (1).

$$\text{But } 2 M_1 K_1^2 = 2 S_1 \beta \left\{ B_1 \frac{(r^4 - r_1^4)}{4} + B_2 \frac{(r^5 - r_1^5)}{5} + B_3 \frac{(r^6 - r_1^6)}{6} + \text{\&c. \&c.} \right\}$$

$M_2 K_2^2 = \dfrac{S_1 \pi h}{2} (r_1^4 - r_2^4)$ See Whewell's Dynamics, page 232.

$$M_2 K_2^2 = \frac{S_2 \pi h}{2} . r_2^4$$

$$\therefore \left\{ 2 M_1 + M_1 + M_2 \right\} K^2 = \frac{4 \pi S_1 h}{p} \left\{ B_1 \frac{(r^4 - r_1^4)}{4} + B_2 \frac{(r^5 - r_1^5)}{5} + \&c. \&c. \right\}$$
$$+ \frac{\pi h}{2} \left(S_1 r_1^4 + (S_2 - S_1) r_2^4 \right) \quad . \; . \; . \quad (2).$$

CHAPTER IV.

PROBLEM I.

Description of the Screw Blade with a Variable Pitch.

If we refer to the description of a conoidal surface, page (3), it will appear there are a great variety of conoidal surfaces, which may be employed to propel vessels through the water; the particular form of each depends upon the nature of the directrix $B\,G$, fig. (3, *Plate*).

That conoidal surface, which is described when the directrix is the common helix, has been used by Mr. Smith; and Professor Woodcroft obtained a patent on the 22nd of March, 1832, for the use of any conoidal surface described, having any other directrix than that of the common helix as adopted by Mr. Smith.

Professor Woodcroft, in his patent, does not attempt to determine the particular form of directrix to be employed in order to obtain the conoidal surface which is the most advantageous for propulsion.

Without offering an opinion which of the two, Mr. Smith's or Professor Woodcroft's, form of conoidal surface is best adapted to the purpose of propelling vessels, I may state my conviction, that a slight advantage gained by Mr. Smith's form of conoidal surface over that employed by Professor Woodcroft, is no proof that the former mode of propulsion is superior to the latter (and *vice versâ*).

The question, I conceive, is as follows:

Every screw blade, the form of which is conoidal, must necessarily have *length, diameter, and directrix, which may be called the elements of the screw blade.*

Hence, it appears, there are three independent elements in the formation of the conoidal surface used to propel vessels. If we take two different directrices, we shall have for each directrix a class of conoidal surfaces, which may be varied at pleasure, by means of the constant parameters which are contained in each directrix. Now, in each class of conoidal surface,

the screw blade may have its elements determined in such a manner, as to be better adapted to propel vessels than any other screw blade of the same class of conoidal surface; therefore, there is the best possible screw blade in each class of conoidal surface, and experiments have hitherto failed to determine whether the best possible screw blade in one class of conoidal surface be better than the best possible screw blade in any other class of conoidal surface.

The form of the conoidal surface may be made to vary in its properties, not only by altering the directrix, or the angle of the screw blade, but also by altering the motion of the generatrix; see page 3. This method of describing surfaces for the screw blade would give rise to another class of surfaces infinite in number, and differing in form from those specified by Professor Woodcroft.

These two modes of variation may be taken simultaneously or separately, to suit the views of the designer of the screw blade.

In cor. (1), prob. (2), chap. (1), it is there stated that in Mr. Smith's screw blade, a line parallel to the directrix is another helix concentric with, and having the same pitch as the directrix.

Now, if we conceive the helices, which are concentric with the directrix, to have a pitch varying according to a given law, or varying by means of a function of their distance from the axis of the screw, we shall obtain a series of surfaces which are not conoidal, but which may have properties favourable to the system of propulsion.

It will be convenient to distinguish the surfaces here described in the following manner:

Constant pitch surface: called Smith's screw blade.

Rising pitch surface: Professor Woodcroft's screw blade.

Rising helix surface.

Any error in the construction of the *constant pitch surface* tends to give the *rising helix surface.*

PROBLEM II.

Geometrical description of the Various Surfaces described in the last Problem.

In fig. (3), (see *Plate*), BC, BF, are radii of the cylinder, and CE its axis, which is at right angles to the lines BC and CF, and consequently perpendicular to the plane BCF or xy. When the line EG moves parallel to the plane xy, and at the same time the extremity of it, G, moves on any line BG whatever, the surface $ECBG$ is called conoidal.

If the line BG, which is called the directrix, be the common helix wrapped on the cylinder, the surface $ECBG$ is the *constant pitch surface,* or Smith's screw blade, generally used in Her Majesty's navy; many of the properties of which are investigated in the foregoing pages.

If the line $B\,G$ be a helix, with a pitch varying according to a given law, that is, a helix the pitch of which is not the same for any two points, however near to each other they may be taken, then the surface $E\,C\,B\,G$ is the *rising pitch surface*, or the screw blade recommended by Professor Woodcroft in his patent of 1832.

If the line $B\,G$ be fixed, and the line $F\,G$, which is the generatrix, moves not parallel to the plane $x\,y$, then another class of surfaces would be described, called twisted surfaces.

If the line $E\,G$ moves in such a manner as to make the line $B'\,G'$ a helix with a constant pitch, but the pitch to vary as the helix approaches to or recedes from the axis $C\,E$, the surface $E\,C\,B\,G$ is the *rising helix surface*.

PROBLEM III.

To Develop the Rising Pitch Surface.

Let $D\,Q$, fig. (3, *Plate*), be the development of the curve, which is the locus traced on the cylinder by the point which is the extremity of the line representing the pitch.

The curve $D\,Q$ may be conveniently called the *pitch director*. Let $B\,S$ be the development of the helix described with a pitch which varies from $B\,D$ to $S\,Q$; this curve depends entirely upon the *pitch director* and the length of the pitch at the points a_2, b_2, &c.

Make $B\,R$ equal to $B\,F$, and draw $T\,U$ parallel to $B\,R$, making $R\,U$ equal to the radius of the cylinder, fig. (1); draw $S\,T$ parallel to $B\,R$, $R\,S$ being the length of the screw blade. Next, to develop the helix $B'\,G'$ at the point B', fig. (3), $B\,S$ representing the development of the helix $B\,G$. Take $R\,W$ equal to $C\,B'$, figs. (1), (3), and (4), and divide $B\,R$ into any convenient number of equal parts at the points a, b, c, &c. &c., from which draw $a\,a_7$, $b\,b_7$, &c. &c., parallel to $B\,D$, cutting the *pitch director* in a_7, b_7, c_7, &c.

Join $B\,a_3$, $B\,b_3$, &c., and draw $W\,V$ parallel to $B\,R$, cutting $B\,a_3$, $B\,b_3$, &c., in a_5, b_5, &c. &c.

Draw $a_2\,a_4$, $b_2\,b_4$, &c., parallel, and $a_5\,a_6$, $b_5\,b_6$, &c., perpendicular to $B\,R$; the intersections a_6, b_6, &c., of the lines $a_2\,a_4$, $b_2\,b_4$, &c., with $a_5\,a_6$, $b_5\,b_6$, &c., trace out the development of the helix at the point B', figs. (1), (3), and (4).

The above method of developing the helix at any point on the screw blade, depends on the property that equal ordinates in the development of the two helices $B\,G$ and $B'\,G'$, fig. (3), have abscissæ which are in the same proportion as $C\,B$ to $C\,B'$.

The length of $B\,S$ and $B\,S'$ may be measured, and placed in the position of $B\,G$ and $B'\,G'$, fig. (4); then the figure $E\,C\,B\,G$, fig. (4), will represent the development of the screw blade, and the area of this figure will be the area of the screw blade. As many helices, in fig. (3), must be developed in

fig. (2), as will enable the draughtsman to draw the curved line $E\,G$, fig. (4), with tolerable accuracy.

The curve $E\,G$, fig. (4), may be called the ***area director;*** it is a hyperbola in Smith's constant pitch surface.

From the above development, and the area director, the area of any conoidal surface may be readily obtained by the draughtsman, with his rule and compasses only.

PROBLEM IV.

To Develop the Constant Pitch and Rising Helix Surfaces.

Let $D\,Q'$, fig. (2, *Plate*), be the helix director, which is used to determine the pitch of a helix, at any distance from the centre of the screw blade.

By referring to problem (2), page 5, we shall see that $B\,G$, fig. (2), is the development of the helix $B\,G$, fig. (3); and, if the pitch were to continue the same, for each concentric helix from $B\,G$ to the axis of the cylinder, $B\,G'$ would be the development of the helix $B'\,G'$ at the distance $B'\,Z'$ from the centre of the cylinder; but, if the pitch does not continue the same for each concentric helix, from $B\,G$ to the axis of the cylinder, then the line $B\,G'$ will represent the development of the helix at the distance $R'\,Z$ from the centre of the cylinder, and $R'\,Q'$ its pitch. To form the area director, which represents the development of the screw blade, both for the ***constant pitch*** and the ***rising helix surfaces,*** a sufficient number of developments $B\,G$, $B\,G'$, &c., &c., must be obtained; after this, the draughtsman will experience little difficnlty in finding the area of the screw blade by means of his rule and compasses.

The constructions in this, and the last problem, are so obvious as to render a demonstration needless.

PROBLEM V.

Having given the Equation to the Helix Director, $D\,Q'$, fig. (2), to find the Equation to the Area Director, $E\,G'\,G$, fig. (4, Plate).

Let $\left.\begin{array}{l} X = B\,R' \text{ in fig. (2)} \\ Y = R'\,Q' \quad \text{,,} \end{array}\right\}$ be the rectangular co-ordinates whose origin is at B.

Let $\left.\begin{array}{l} x = C\,D \text{ in fig. (4)} \\ y = D\,P \quad \text{,,} \end{array}\right\}$ be the rectangular co-ordinates whose origin is at C.

$$\text{Let } Y = \mathrm{f}\,(X) \quad . \quad . \quad . \quad . \quad . \quad (1),$$

be the equation to the helix director.

$$I'\,B = h, \text{ the length of the screw blade.}$$

$\therefore$ by similar triangles, $B\,G' : B\,Q' :: h : R'\,Q'$.

$$\text{Or, } y : \sqrt{X^2 + Y^2} :: h : Y$$

$$\therefore y = h\sqrt{1 + \left(\frac{X}{Y}\right)^2} \quad . \quad . \quad . \quad (2).$$

But $B\,R'$, fig. (2), is equal to $2\,\pi \,.\, C\,D$, fig. (4), because $B\,R'$ is the circumference of a circle whose radius is $C\,D$.

$$\therefore X = 2\,\pi\,x$$

$$\text{Therefore, } Y = \mathrm{f}\,(2\,\pi\,x).$$

Substitute these values in (2), we shall have

$$y = h\sqrt{1 + \left(\frac{2\,\pi\,x}{\mathrm{f}\,(2\,\pi\,x)}\right)^2} \quad . \quad . \quad . \quad (3),$$

which is the equation to the *area director*.

Cor. (1).—By taking various curves for the *helix director*, equation (3) will give the equation to the *area director*.

When the helix director is a straight line $D\,D$ parallel to $B\,B$, which is the case in Smith's screw blade, we shall have

$$\mathrm{f}\,(2\,\pi\,x) = h$$

$$\therefore y = \sqrt{h^2 + 4\,\pi^2\,x^2} \quad . \quad . \quad . \quad (4).$$

The equation to the *area director*, which in this case is a hyperbola.

PROBLEM VI.

Having given the equation to the *area director*, $E\,G$, fig. (4), to find the equation to the helix director, $D\,Q$, fig. (2),

Let $\left.\begin{array}{l} X = C\,D \text{ in fig. (4)} \\ Y = D\,P \quad\quad ,, \end{array}\right\}$ be the rectangular co-ordinates whose origin is at C.

Let $\left.\begin{array}{l} x = B\,R' \text{ in fig. (2)} \\ y = R'\,Q' \quad\quad ,, \end{array}\right\}$ be the rectangular co-ordinates whose origin is at C.

Also let

$$Y = \mathrm{f}\,(X) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1),$$

be the equation to the *area director*.

$I'\,B = h$, fig. (2), is the length of the screw.

By similar triangles we shall have

$$B\,G' : B\,Q' :: G\,F : R'\,Q', \text{ fig. (2)},$$

$$\therefore Y : \sqrt{x^2 + y^2} :: h : y.$$

$$\text{Or, } Y = h\sqrt{1 + \frac{x^2}{y^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

Again, because $B\,R' = 2\,\pi\,.\,C\,D$, we shall have

$$x = 2\,\pi\,X \quad \therefore \; X = \frac{x}{2\,\pi}$$

And from (1) we have, $Y = \mathrm{f}\left(\frac{x}{2\,\pi}\right)$. Substitute this value in equation (2), we shall have

$$\mathrm{f}\left(\frac{x}{2\,\pi}\right) = h\sqrt{1 + \frac{x^2}{y^2}}$$

From which we obtain

$$y = \frac{h\,x}{\sqrt{\mathrm{f}\left(\frac{x}{2\,\pi}\right)^2 - h^2}} \quad . \quad . \quad . \quad (3),$$

which is the equation to the helix director.

By taking various curves for the area director, equation (3) will give the equation to the helix director.

PROBLEM VII.

To find the Equation to the Rising Pitch Surface.

Let x, y, z, be the rectangular co-ordinates of a point on the surface $E\,C\,B\,G$, fig. (3); $B\,G$ is the helix described in problem (3), chap. (4); $C\,B$, $C\,E$, are the axes of x and z, perpendicular to which is the axis of y, origin at C.

If we call β' the angle of the plan corresponding to the height z, we shall have

$$z = \mathrm{f}\,(r\,\beta') \quad . \quad . \quad . \quad . \quad . \quad (1).$$

Because the height on the cylinder varies as some function of the circular arc $B\,F$.

But in every conoidal surface we shall have

$$\tan.\,\beta' = \frac{y}{x};\ \text{or,}\ \beta' = \tan.^{-1}\left(\frac{y}{x}\right)$$

Therefore, equation (1) becomes

$$z = \mathrm{f}\left\{ r\tan.^{-1}\left(\frac{y}{x}\right) \right\} . \quad . \quad (2),$$

which is the equation to the *rising pitch surface* of Professor Woodcroft.

Equation (1) is the equation to the curve $B\,S$ in fig. (2), where the z and $r\,\beta'$, in equation (1), are the y and x of the curve $B\,S$, in fig. (2).

When the function $\mathrm{f}\,(r\,\beta')$ is represented by $r\,\beta'$, we shall obtain the equation to the *constant pitch surface*.

PROBLEM VIII.

To find the Equation to the Helix Pitch Surface.

Using the same notation as in the last problem, we shall have, by referring to prob. (2), page (4),

$$y = x \tan. \left(\frac{z}{r \tan. A} \right)$$

From which we obtain

$$z = r \tan. A \tan.^{-1} \left(\frac{y}{x} \right)$$

$$= \frac{p}{2 \pi} . \tan.^{-1} \left(\frac{y}{x} \right) \quad . \quad . \quad (1).$$

Now if p be represented by a function of $2 \pi x$, we shall have

$$p = f \left(2 \pi \sqrt{x^2 + y^2} \right) \quad . \quad . \quad (2).$$

Therefore equation (1) becomes

$$z = \frac{f(2 \pi \sqrt{x^2 + y^2})}{2 \pi} \tan.^{-1} . \left(\frac{y}{x} \right) \quad . \quad (3),$$

which is the equation to the *helix pitch surface.*

The p and $2 \pi \sqrt{x^2 + y^3}$ of equation (2) are the y and x of the helix director, fig. (2).

Scholium.—The three surfaces are represented by the equations

$$\left.\begin{array}{ll} z = \frac{p}{2 \pi} \tan.^{-1} \left(\frac{y}{x} \right) & \text{in the } \textit{constant pitch surface.} \\ z = f \left\{ r \tan.^{-1} \left(\frac{y}{x} \right) \right\} & \text{in the } \textit{rising pitch surface.} \\ z = \frac{f(2 \pi \sqrt{x^2 + y^2})}{2 \pi} \tan.^{-1} \left(\frac{y}{x} \right) & \text{in the } \textit{helix pitch surface.} \end{array}\right\} \quad . \quad (4).$$

PROBLEM IX.

To find the Equation to the Curve described by a Point in the Surface of the Screw Blade.

Let the axis of x pass through the point in the screw blade

so that x_1, 0, 0, are the rectangular co-ordinates of the given point when at rest,

and x, y, z, are the rectangular co-ordinates at the end of the time t.

Put w = angular velocity of the screw blade.
And v = velocity of the vessel.

Then $x_1\ w$ = angular velocity of the given point.

From which we shall obtain, by the conditions of the problem,

$$d x^2 + d y^2 = x_1^2 w^2 d t^2, \text{ and } d z^2 = v^2 d t^2 \quad . \quad . \quad (1).$$

From these equations we obtain

$$d x^2 + d y^2 = \frac{x_1^2 w^2}{v^2} . d z^2 \quad . \quad . \quad (2).$$

By the question $x^2 + y^2 = x_1^2$ . . (3).

Differentiate this equation, and we shall have

$$d x = - \frac{y}{x} d y \quad \therefore d x^2 = \frac{y^2}{x^2} d y^2$$

Substitute this in equation (2), we have

$$\frac{d y}{\sqrt{x_1^2 - y^2}} = \frac{w}{v} d z. \text{ And } - \frac{d x}{\sqrt{x_1^2 - y^2}} = \frac{w}{v} d z$$

Integrate these equations, we shall obtain

$$y = x_1 \sin. \left(\frac{u}{v} z\right) . \text{ And } x = x_1 \cos. \left(\frac{w}{v} z\right) \quad . \quad . \quad (4).$$

These equations, together with equation (3), show that the path described by a point on the surface of the screw blade is a helix whose angle is equal to $\tan.^{-1}\left(\frac{v}{w x^1}\right)$, and pitch equal to $\frac{2 \pi v}{w}$.

Cor. (1).—If we transfer the origin of co-ordinates along the axis of z to a point z' below the present origin, and the axes of y and z through an angle $\tan.^{-1}\left(\frac{y'}{x'}\right) = \theta$, we shall have

$$\left.\begin{aligned} x &= X \cos. \theta + Y \sin. \theta \\ y &= Y \cos. \theta - X \sin. \theta \\ z &= Z - z' \end{aligned}\right\} \quad . \quad . \quad (5),$$

where X, Y, Z, are the rectangular co-ordinates of any point in the helix; substitute these values in equations (4), and we shall have

$$X \cos. \theta + Y \sin. \theta = x_1 \cos. \frac{w}{v} (Z - z')$$

$$Y \cos. \theta - X \sin. \theta = x_1 \sin \frac{w}{v} (Z - z')$$

From these equations we shall have

$$\left.\begin{aligned} Y &= x_1 \left\{ \cos. \theta . \sin. \frac{w}{v} (Z - z') + \sin. \theta \cos. \frac{w}{v} (Z - z') \right\} \\ X &= x_1 \left\{ \cos. \theta . \cos. \frac{w}{v} (Z - z') - \sin. \theta . \cos. \frac{w}{v} (Z - z') \right\} \\ & X^2 + Y^2 = x_1^2 \end{aligned}\right\} \quad (6).$$

Since $X^2 + Y^2 = x_1^2 = y'^2 + x'^2$

And $\text{Cos.}\, \theta = \dfrac{x'}{\sqrt{x'^2 + y'^2}}$ and $\sin.\, \theta = \dfrac{y'}{\sqrt{x'^2 + y'^2}}$

$$\left.\begin{aligned} Y &= x' \sin. \frac{w}{v}(Z - z') + y' \cos. \frac{w}{v}(Z - z') \\ X &= x' \cos. \frac{w}{v}(Z - z') - y' \sin. \frac{w}{v}(Z - z') \\ X^2 &+ Y^2 = x'^2 + y'^2 \end{aligned}\right\} \quad . \quad . \quad (7).$$

PROBLEM X.

To find the Normal Velocity at a point on the Screw Blade having a given Equation.

Let P be a point on the surface of the screw blade, whose rectangular co-ordinates are x, y, and z, respectively.

The origin and co-ordinate planes are the same as described in page (4), problem (2).

Through P draw a cylinder whose axis is the axis of Z, and whose radius is $\sqrt{x^2 + y^2}$.

From the point P, parallel to the plane of $x\,y$, draw a tangent T to the cylinder.

The angular velocity in the direction of the tangent T is expressed by

$$w\sqrt{x^2 + y^2} \quad . \quad . \quad . \quad . \quad (1).$$

where w is the angular velocity at a unit of distance from the axis of z.

Let ϕ be the angle which a normal to the screw blade at P makes with the axis of z; and Q be the point at which this normal intersects the plane of $x\,y$. Let R be the projection of P on the plane of $x\,y$; then $R\,Q$ will be the projection of the normal $P\,Q$ on the plane $x\,y$.

The equations to $P\,Q$ are, putting $\dfrac{dz}{dx} = p$, and $\dfrac{dz}{dy} = q$

$$x' - x + p(z' - z) = 0, \text{ and } y' - y + q(z' - z) = 0 \quad . \quad . \quad . \quad (2),$$

where x', y', z', are the current co-ordinates of the normal. See Gregory's Solid Geometry, page 135.

To find where the normal pierces the plane of $x\,y$ we must make $z' = 0$ in equations (2).

$$\therefore x' - x = p\,z, \text{ and } y' - y = q\,z \quad . \quad . \quad . \quad (3).$$

$$\therefore R\,Q = \sqrt{(x' - x)^2 + (y' - y)^2} = z\sqrt{p^2 + q^2}$$

$$\text{and } P\,Q = \sqrt{z^2 + z^2 p^2 + z^2 q^2} = z\sqrt{1 + p^2 + q^2}$$

$$\text{and } \cos.\, \phi = \frac{z}{P\,Q} = \frac{1}{\sqrt{1 + p^2 + q^2}}$$

Put $\phi' =$ the angle which $R\,Q$ makes with the projection of T on the plane of $x\,y$.

Produce $Q\,R$ to meet the axis of x in S.

$$\therefore \text{Cos.} \left(\frac{\pi}{2} - \phi'\right) = \sin.\phi' = \cos.(S - C) = \cos. S \cos. C + \sin. S \sin. C,$$

$$\text{Sin.} \left(\frac{\pi}{2} - \phi'\right) = \cos.\phi' = \sin.(S - C) = \sin. S \cos. C - \cos. S \sin. C.$$

$$\text{But } y' - y : x' - x :: y : (x - A\,S)$$

$$\therefore x - A\,S = \frac{p\,z\,y}{q\,z} = \frac{p\,y}{q}$$

$$\text{And } \cos. S = \frac{x - A\,S}{\sqrt{y^2 + (x - A\,S)^2}} = \frac{p}{\sqrt{p^2 + q^2}}$$

$$\text{Sin. } S = \frac{q}{\sqrt{p^2 + q^2}}$$

$$\text{Cos. } C = \frac{x}{\sqrt{x^2 + y^2}}, \text{ and } \sin. C = \frac{y}{\sqrt{x^2 + y^2}}$$

$$\therefore \sin.\phi' = \frac{x\,p + y\,q}{\sqrt{x^2 + y^2}\sqrt{p^2 + q^2}}$$

$$\text{Cos. } \phi' = \frac{x\,q - y\,p}{\sqrt{x^2 + y^2}\sqrt{p^2 + q^2}}$$

It remains, therefore, to decompose the velocity along the tangent T, and the velocity along the axis of z, into the direction of the normal, in order to obtain the normal velocity V_1.

$$\therefore V_1 = w\sqrt{x^2 + y^2}\,.\ \text{Cos.}\,\phi' \sin.\phi - v \cos.\phi$$

where $v =$ the velocity of the vessel.

Substituting the values of cos. ϕ', &c., in the above equation we shall have

$$V_1 = \frac{w\,(x\,q - y\,p) - v}{\sqrt{1 + q^2 + p^2}} \quad . \quad . \quad (4),$$

which is the normal velocity at any point whose rectangular co-ordinates are x, y, and z respectively.

The values of p and q, which are functions of x and y, must be obtained from the equation to the surface.

PROBLEM XI.

To find the Forces acting on the Blade of the Screw.

Adopting the same notation as in the last problem, we shall have

$n\,F\,(V_1)\,d\,m =$ the accelerating force in the direction of the normal . (1),

where $d\,m$ represents the element of the screw blade at a point whose rectangular co-ordinates are x, y, z, and n a constant quantity.

The normal accelerating force must be resolved in two directions; viz.,

in the direction of the axis of z, and in the direction of the tangent T drawn from the point P: the former gives motion to the vessel, and the latter resists the motion of rotation of the screw.

Hence $n\,F(V_1)\cos.\varphi\,d\,m$; $n\,F(V_1)\sin\phi\,.\cos.\varphi'\,d\,m$, and $n\,F\,(V_1)\sin.\phi\sin\varphi'\,d\,m$, are the resolved forces parallel to the axis of z, tangent T, and radius at P parallel to the plane of $x\,y$ respectively.

Substituting the values of $\cos.\varphi$, $\sin.\phi$, &c. &c., as given in the last problem, and observe that

$$d\,m = \sqrt{1 + p^2 + q^2}\,.\,d\,x\,d\,y$$

we shall have

$n\iint F(V_1)\,d\,x\,d\,y$ = accelerating force parallel to z . . . (2).

$n\iint F(V_1)\,(x\,q - y\,p)\,d\,x\,d\,y$ = moment of accelerating force to turn the screw round . . . . (3).

$n\iint F(V_1)\dfrac{x\,p + y\,q}{\sqrt{x^2+y^2}}\,d\,x\,d\,y$ = accelerating force in the direction of the radius at P . . . . (4).

Put Υ_ϖ = the number of units of moments of force to turn the screw round, when ϖ pounds pressure, as measured by the indicator, are applied to the piston.

And R'_ϖ = the accelerating force of resistance to the motion of the vessel, when ϖ pounds pressure are applied to the piston.

By dynamics we have $\dfrac{d\,w}{d\,t}$ = angular accelerating force of the screw. See Dr. Whewell's Dynamics, part ii., pages 185, 118.

$$\therefore \frac{d\,w}{d\,t} = \frac{\Upsilon_\varpi}{2\,I} - 2\,n\iint F(V_1)\,(x\,q - y\,p)\,d\,x\,d\,y \quad . \quad . \quad (5).$$

$$\text{And } \frac{d\,v}{d\,t} = 2\,n\iint F(V_1)\,d\,x\,d\,y - R'_\varpi \quad . \quad . \quad . \quad . \quad (6).$$

where I is the moment of inertia of the screw blade.

The double integrals in equations (5) and (6) must be integrated through the whole extent of the surface of the screw blade.

When a uniform motion, both in the screw and the vessel, is obtained, the accelerating force is nothing. Therefore, equations (5) and (6) will become

$$\frac{\Upsilon\varpi}{2\,I} = 2\,n\iint F(V_1)\,x\,q - y\,p)\,d\,x\,d\,y \quad . \quad . \quad . \quad (7).$$

$$R'_\varpi = 2\,n\iint F(V_1)\,d\,x\,d\,y \quad . \quad . \quad . \quad . \quad . \quad (8).$$

Hence, all the important equations which are obtained in notes (1) *and* (2), *chap.* ii., *will be true for every screw blade whose surface is determined from the equation*

$$x\,q - y\,p = c, \text{ a constant quantity} \quad . \quad . \quad (9).$$

Therefore, equations (7) and (8) become

$$\frac{\Upsilon_{\varpi}}{2\,I} = 2\,n\,C\iint F(V_1)\,d\,x\,d\,y\,. \quad . \quad (10).$$

$$R'_{\varpi} = 2\,n\iint F(V_1)\,d\,x\,d\,y \quad . \quad . \quad (11).$$

$$\therefore\ \frac{\Upsilon_{\varpi}}{2\,I} = C\,R'_{\varpi} \quad . \quad . \quad . \quad . \quad (12),$$

which is true for every screw blade whose surface is determined in conformity with the equation (9).

Equation (9) is integrated in the next problem.

Cor. (1).—The best possible surface to be used for a screw blade must be determined by making the indefinite integral

$$\iint F(V_1)\ d\,x\ d\,y \text{ a maximum.}$$

Hence, R'_{ϖ} must be a maximum.

Therefore, if the power be constant in equation (12), $I\,C$ must be a minimum.

In the common screw we shall have

$I\,P$ to be a minimum.

The best velocity with the same power will be obtained by the constant pitch surface when the *rectangle of the pitch and moment of inertia of the blades is a minimum.*

PROBLEM XII.

To find the Condition that the Screw Blade shall have a Surface of Vanishing Pressure.

When the screw blade is on the *surface of vanishing pressure,* the normal velocity of the screw blade must vanish for every point on it.

Hence, by referring to problem, equation (4), we must have

$$w\left(x\,\frac{d\,z}{d\,y} - y\,\frac{d\,z}{d\,x}\right) = v \quad . \quad . \quad . \quad . \quad (1).$$

$\frac{d\,z}{d\,y}$ and $\frac{d\,z}{d\,x}$ are both functions of x and y, which must be obtained from the equation to the screw blade.

Therefore, every surface whose equation gives

$$x \frac{d z}{d y} - y \frac{d z}{d x} = C \quad . \quad . \quad (2).$$

for every point, throughout its whole extent, has a surface of vanishing pressure.

Equation (2) can be integrated in the following manner:

Put $y\,d\,y = d\,y'$, and $x\,d\,x = d\,x'$.

$$\therefore \frac{d z}{d y'} - \frac{d z}{d x'} = \frac{C}{x\,y}$$

$$\text{But } y' = \frac{y^2}{2}, \text{ and } x' = \frac{x^2}{2}$$

$$\therefore \frac{d z}{d y'} - \frac{d z}{d x'} = \frac{C}{2\sqrt{x'\,y'}}$$

$$\text{Or } z = \left\{\frac{d}{d y'} - \frac{d}{d x'}\right\}^{-1} \cdot \frac{C}{2\sqrt{x'\,y'}}.$$

Integrate this with respect to y, and suppose $\frac{d}{d x}$ to be constant, we shall have

$$z = E^{y' \frac{d}{d x'}} \int d\,y'\,E^{-y' \frac{d}{d x'}} \cdot \frac{C}{2\sqrt{x'\,y'}}$$

See Gregory's Examples, page 244.

$$\therefore z = E^{y' \frac{d}{d x'}} \int \frac{C\,d\,y'}{2\sqrt{(x' - y')\,y'}}$$

$$= E^{y' \frac{d}{d x'}} \left\{ \frac{C}{2} \text{ vers.}^{-1} \left(\frac{2 y'}{x'}\right) + \frac{C}{2} \phi\,(x') \right\}$$

$$\therefore z = \frac{C}{2} \left\{ \text{vers.}^{-1} \left(\frac{2 y'}{x' + y'}\right) + \phi\,(x' + y') \right\}$$

$$= \frac{C}{2} \left\{ \text{vers.}^{-1} \left(\frac{2 y^2}{x^2 + y^2}\right) + \phi\,(y^2 + y^2) \right\} . \quad . \quad (3).$$

where ϕ is an arbitrary function.

Equation (2) may be integrated in a different manner from the above.

Lagrange has shown that, if we can obtain two integrals,

$$F\,(x, y, z) = a, \text{ and } F_1\,(x, y, z) = b$$

from the equations

$$d\,y + \frac{x}{y}\,d\,x = 0;\ d\,z + \frac{C}{y}\,d\,x = 0, \text{ and } d\,z - \frac{C}{x}\,d\,y = 0$$

$$\text{we shall have, } b = \phi\,(a) \quad . \quad . \quad . \quad (4).$$

From the first we have

$$y^2 + x^2 = a \quad \therefore x = \sqrt{a^2 - y^2}$$

$$\text{Or} \therefore dz - \frac{C\,dy}{\sqrt{a^2 - y^2}} = 0.$$

Integrate this equation, and we shall have

$$z - C \sin.^{-1}\left(\frac{y}{a}\right) = b.$$

Hence $z - C \sin.^{-1}\left(\frac{y}{a}\right) = \phi\,(x^2 + y^2)$

$$\therefore z = C \sin.^{-1}\left(\frac{y}{\sqrt{x^2 + y^2}}\right) + \phi\,(x^2 + y^2)$$

$$\text{Or } z = C \tan.^{-1}\left(\frac{y}{x}\right) + \phi\,(x^2 + y^2) \quad . \quad . \quad . \quad (5).$$

This is also the surface which will give the same equations as those obtained in notes (1) and (2), chap. ii.

Scholium.—Professor Woodcroft was the first to introduce the screw blade with a rising pitch, that is, with a pitch which varies at every point along the extremity of the screw blade as described in one of the foregoing problems.

He also has the merit of being the first to introduce another variation of considerable importance, which is to turn the screw blades through an angle in opposite directions. This variation, which is an admirable expedient to overcome a great difficulty, he thinks will produce an effect, in the velocity of the vessel, somewhat similar to that produced by increasing or diminishing the pitch in the constant pitch surface.

This variation is not useful in the screw propeller with a rising pitch only, but also in the one with a constant pitch. Suppose the blade of the screw to have its extreme helix making an angle of twenty degrees, and Professor Woodcroft's apparatus to be applied in such a manner as to turn this helix through every angle to fifteen degrees; then, if we could persuade ourselves that the screw blade thus moved from its first position to its second fully coincided with a screw blade whose angle is fifteen degrees, the variation to which we allude would be a great improvement in the art of screw propulsion. If the angle through which this variation takes place be small, the screw blade, in its second position, will nearly coincide with a screw blade having the same angle; but generally this coincidence does not take place, and the relation of the screw blade to the axis of rotation in the one case is not the same as in the other.

The effect which is produced on the equation of the screw blade by this variation, is obtained by placing, in the equation of the screw blade, $z + y \tan.\,\theta$ instead of z, and $y - z \tan.\,\theta$ instead of y, where θ is the angle through which the blades of the screw have been turned.

Hence, we shall have, after obvious reductions, the following relation:

$$z = \frac{p}{2\pi} \tan.^{-1} \left(\frac{y}{x}\right) - \frac{p}{2\pi} \tan.^{-1} \left\{ \frac{x\,z \tan.\theta}{yz \tan.\theta - x^2 - y^2} \right\} - y \tan.\theta,$$

the equation to the *constant pitch surface.*

By formula (5), page 37, we shall have the accelerating force, on the bow and other parts of a vessel, inversely proportional to the rectangle of the pitch and moment of inertia of the screw blade, when the motion of the screw and vessel is uniform, and the force constant. Now, the velocity of the vessel is increased when the accelerating force is increased, or the rectangle of the pitch and moment of inertia is decreased.

Hence, if we suppose the screw blade to coincide in its second position with a screw blade of a less angle, and the moment of inertia not to be practically altered, which is really the case in practice, then the velocity of the vessel will evidently increase as the pitch is diminished between certain limits.

Therefore, a large pitch or angle must be used to set the vessel in motion, and when that becomes uniform, as it will, a smaller pitch will be advantageous.

The all important subject of resistance of fluids prevents me from entering further into the discussion of the class of surfaces which have been used for propelling vessels by Professor Woodcroft.

I may, however, be permitted to say, that neither Smith's nor Woodcroft's surfaces, nor any other which have been used, are the best possible surfaces which may be adopted. The best possible surface must be determined from

$$U = \iint F\,(V_1)\; d\,x\; d\,y \text{ being a maximum.}$$

$$\text{Or } U = \iint F \left\{ \frac{w\,(x\,q - y\,p) - v}{\sqrt{1 + q^2 + p^2}} \right\} d\,x\; d\,y \quad \text{a maximum.}$$

Now, if we suppose the resistance of fluids to vary as the square of the normal velocity, which is the usual theory, we shall have

$$U = \iint \frac{\left\{ w\,(x\,q - y\,p) - v \right\}^2}{1 + q^2 + p^2}\, d\,x\; d\,y \quad \text{a maximum.}$$

The discussion of this indefinite integral will be given in another place.

NOTES ON CHAPTER IV.

Note (1).

From cor. (3), prob. (1), chap. (1), we have the length of a helix at r distance from the axis of the screw,

$$L = \frac{h}{p} \sqrt{4 \pi^2 r^2 + p^2}$$

where L is the length of the helix whose pitch and length are p and h respectively.

From this equation we readily obtain

$$p = \frac{2 \pi h r}{\sqrt{L^2 - h^2}} \quad . \quad . \quad . \quad (1).$$

Hence, by measuring the length of a helix, at r distance from the axis, this formula will give the pitch of the screw.

But $r \beta = \sqrt{L^2 - h^2}$, where β is the angle of the plan,

$$\therefore p = h \cdot \left(\frac{2 \pi}{\beta}\right) \quad . \quad . \quad . \quad (2).$$

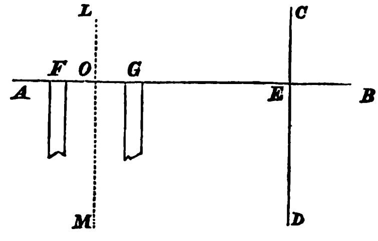
Suppose we take $\beta = \frac{\pi}{6}$, or, which is the same thing, take the angle of the plan to be *thirty degrees*; then equation (2) will become

$$p = 12\, h \quad . \quad . \quad . \quad (3).$$

Now if h be taken in inches, through an amplitude of thirty degrees, then the pitch will be measured by the same number of feet.

On this principle an instrument, called the *Pitch-compass*, has been invented by the author, with a view to facilitate the admeasurement of the pitch of the screw propeller.

By this instrument the pitch of the screw, at any distance from its axis, can be readily ascertained; and I think it right to state, that more labour, with greater accuracy, can be performed by one man in a few minutes with the Pitch-compass, than by three men in several hours by the old methods.



The Pitch-compass is very simple in its construction and use, as the following description and application to practice will show:

FG is a section of the boss of the screw, and LM its axis: AB is a beam compass, whose centre O is the centre of the boss through which the axis of the screw blade passes.

The beam compass, $A\,B$, which is made sufficiently long to extend from the axis LM to the extremity of the screw blade, is divided into feet and inches, and constructed in such a manner that it will turn round the axis $L\,M$, fixed on a graduated circular disk, with perfect freedom.

CD is a brass rod, graduated into inches and tenths of inches, placed neatly in a groove which is fixed firmly to a slide which moves freely from *B* to *O*. The rod is allowed to move in the groove in the direction of *C D*, at right angles to *A B*; and the slide and brass rod can be fixed in any position by means of clamp screws.

Suppose it to be required to measure the pitch of the screw blade at a distance of *eight feet* from the axis of the screw. The slide *E* is then fixed by a clamp screw, at a distance *O E*, *eight feet* from the centre *O*: then the height *E D* of the beam compass from the screw blade is observed to be *seven inches*. After this, the beam compass *A B* is moved gently round through an angle of *thirty degrees*, marked on the graduated disk on which the centre *O* is fixed, while the slide and brass rod remain fixed; the brass rod is then lowered so that *D* again touches the blade of the screw, and the height *E D* is now observed to be *twenty-five inches*.

The difference in the height of the beam compass, *A B*, from the screw blade in the two positions is *eighteen inches*; therefore, the pitch of the screw blade is *eighteen feet*.

A Pitch-compass has been constructed by the author for the use of the Master-shipwright of Her Majesty's Dockyard, Portsmouth; and Andrew Murray, Esq., Chief-Engineer of Her Majesty's Dockyard, Portsmouth, has made an admirable one, for his use in the public service, on the same principle as the above.

By means of the *Pitch-compass* the screw propeller for the *Dauntless*, whose diameter is 14 feet $8\frac{3}{4}$ inches, was measured and found to be

Radius.	Pitch.
7′.. $4\frac{3}{8}$″	16′.. 4″
5..9	16.. $7\frac{1}{2}$
5..0	16.. $9\frac{1}{2}$
4..0	16..11
3..0	17.. 2

Hence the pitch of the screw increases from the extremity of the blade to the axis.

NOTE (2).

If we refer to equation (4), page 51, we shall have

$$T'^2_x = \frac{h\,n\,w\left(1 + \sqrt{\frac{c'}{c}}\right)}{2\,\delta} \cdot \frac{(r - x)^3\,(x + 2\,r)}{\sqrt{\beta^2\,x^2 + h^2}}$$

But, since $\beta = \frac{2\,\pi\,h}{p}$, we shall have

$$T'^2_x = \frac{p\,n\,w\left(1 + \sqrt{\frac{c'}{c}}\right)}{2\,\delta} \cdot \frac{(r - x)^3\,(x + 2\,r)}{\sqrt{4\,\pi^2\,x^2 + p^2}} \quad . \quad . \quad (1).$$

Now, if we take the thickness of the screw blade adjacent to the boss of the screw to be a given quantity, we shall obtain

$$T'^2_{r_1} = \frac{p\,n\,w\left(1 + \sqrt{\frac{c'}{c}}\right)}{k^2\,\delta} \cdot \frac{(r - r_1)^3\,(r_1 + 2\,r)}{\sqrt{4\,\pi^2\,r_1^2 + p^2}}$$

$$\therefore\ T'^2_x = \frac{T'^2_{r_1}\sqrt{4\,\pi^2\,r_1^2 + p^2}}{(r - r_1)^3\,(r_1 + 2\,r)} \times \frac{(r - x)^3\,(x + 2\,r)}{\sqrt{4\,\pi^2\,x^2 + p^2}} \quad . \quad . \quad . \quad (2).$$

The dimensions of the screw used in the *Dauntless*, built by John Fincham, Esq., Portsmouth Dockyard, are as follows:

	Ft. in.		Ft. in.		Ft. in.
Length,	3 .. 0;	radius,	7 .. 3;	and pitch,	17 .. 3.

And the thickness of the screw blade, perpendicular to its surface, is $5\frac{1}{4}$ inches at one foot from the axis of the screw.

By formula (2) we obtain the thickness of the blade; at

1	foot distance	from the axis,	the thickness is	5·25	inches.
2	„	„	„	3·87	„
3	„	„	„	2·64	„
4	„	„	„	1·67	„
5	„	„	„	·91	„
6	„	„	„	·34	„

The actual dimensions of the screw blade are, at

1	foot distance	from the axis,	the thickness is	5·25	inches.
2	„	„	„	3·625	„
3	„	„	„	2·875	„
4	„	„	„	2·25	„
5	„	„	„	1·625	„
6	„	„	„	1·375	„
7	„	„	„	1·125	„

In consequence of the indefinite state of our knowledge respecting the quantities n, c', c, and δ, in formula (3), page 51, it is difficult to determine, other than the form, of the function $F(x)$, to satisfy both conditions expressed in page 46. The first condition, which is that the blade of the screw must not break in one place in preference to another, is completely determined by formula (2).

NOTE (3).

On the Vertical Oscillations of Vessels, resulting from the Action of the Screw Propeller.

When the screw propeller, consisting of two equal blades fixed on opposite sides of the axis, is wholly immersed, its uniform angular motion has no tendency either to raise or depress the stern of the vessel.

By referring to formula (4), page 63, we see that

$G = n \iint F(V_1) \frac{x\,p + y\,q}{\sqrt{x^2 + y^2}}\, d\,x\, d\,y$ expresses the accelerating force in the direction of the radius, or in the direction at right angles to the axis of the screw, where n is a constant quantity depending on the density of the water, which is the same at all depths, if we suppose water to be incompressible, which is certainly true at all ordinary depths.

In consequence of the screw consisting of two blades fixed in opposite directions, there will be another force, G, acting in an opposite direction to the former; the resultant of the two forces will be nothing; therefore, the only effect which is produced by these two forces is, a compression of the screw blades in the direction of the diameter.

When the blades of the screw propeller are not wholly immersed during a part of their revolution, then their uniform angular velocity has a tendency to raise the stern of the vessel.

In this case the integral G becomes G', which is a less quantity than G, and the resultant of these two forces is $G - G'$ when the blades are vertical.

This latter case frequently occurs in practice, and if the time of oscillation of the vessel and the half revolution of the screw propeller be synchronical, the amplitude of the oscillations of the vessel will be considerable, and their effect on the timbers of the vessel will be injurious to the stability of the structure.

If the screw propeller, consisting of three equal blades fixed on the sides of the axis, so that the angle between each two is the same, be wholly immersed, then its uniform angular motion will cause the axis to describe a curve which is called a Tractory.

In this case there are three equal integrals, each of which may be called G, which represents the resultant of all the resolved forces in the direction of the diameter of the screw blade; these equal forces act in directions which make an angle of 120° between each two, and pass through the same point; hence their resultant is $G(\sqrt{3} - 1)$, which is constant when the vessel and screw have attained their uniform motion.

This resultant is constantly acting upon the blade of the screw, to produce oscillations, with an invariable intensity; but its direction is continually changing, making a complete revolution during each revolution of the screw on its axis; therefore, the oscillations of the vessel produced by a three bladed screw are different from those produced by a two bladed screw; in the former case there are horizontal oscillations, which produce an effect to twist the timbers of which the vessel is formed, and thereby materially injure its permanency.

These oscillations, which are destructive to the vessel, can be avoided by making one blade a little longer than the other two, in such a manner that the resultant of the three resolved forces in the direction of the diameter may vanish.

It will be seen from the above reasoning that the vertical oscillations of the stern of the vessel,—which are now occupying the attention of practical men, not only on account of their destructive tendency, which is considerable, and can be observed only through a long series of years, but also on account of the uneasy motion which they impart to that portion of the vessel occupied by the captain and other officers,—are not produced by one blade of the screw revolving in deeper water than the other, but from the resolution of the forces on the screw blade in the direction of its diameter. The water is no more solid, in a practical sense, at the depth of twenty feet than it is at one foot; and if the blades of the screw be just covered with water, they are then working in water as solid as if they were immersed to a greater depth.

In Smith's screw (see page 59) we shall have

$$z = \frac{p}{2\pi} \tan.^{-1}\left(\frac{y}{x}\right)$$

$$\therefore \frac{dz}{dx} = -\frac{p}{2\pi}\,\frac{y}{\sqrt{x^2+y^2}}, \text{ and } \frac{dz}{dy} = \frac{p}{2\pi}\,\frac{x}{\sqrt{x^2+y^2}}$$

$$\therefore x\frac{dz}{dx} + y\frac{dz}{dy} = 0$$

Hence, independently of the peculiar action of the water on the blade of the screw, the resultant of the resolved forces to produce oscillations at the stern of the vessel in Smith's screw entirely vanishes, and there can be no oscillations produced excepting those arising from the screw blade not being perfect in its form. The Pitch-compass, described in page 68, has detected the inaccuracy, in this respect, of several screws to which it has been applied.

One distinguishing feature, therefore, in Smith's screw blade is, that its peculiar form renders the resolved forces, in the direction of the diameter, zero; consequently there can be no pressure upon the blade of the screw in the direction of its radius, to produce vertical oscillations in the vessel.

This condition is peculiar to the form of Smith's screw blade; and its importance results from the fact, that it is a means of obviating the inconvenient and injurious effects which would be produced on the after part of the vessel by constant oscillations, however small might be their amplitude.